Energy and Power
Objective of Lecture

- Describe the mathematical relationships between power and energy.
  - Chapter 4.1
- Relate power to voltage and current.
  - Chapter 4.2
- Explain the differences between positive and negative power using passive sign convention.
  - Chapter 4.4
- Discuss issues concerning power supplies and batteries.
  - Chapter 4.5
Electrical Power

- Power ($P$) is the amount of energy ($W$) required during a length of time ($\Delta t$) to do work or the time-rate of change in energy.
- In our case, the work is forcing electrons to move in and out of the various components in the circuit.

\[ P = \frac{W}{\Delta t} \quad \text{or} \quad p = \frac{dw}{dt} \]
Energy

Energy is the capacity to do work.

\[ W = P \Delta t \]
\[ w = \int_{t_1}^{t_2} p \, dt \]
Units

- Units for power is W (Watts), which is a Joule per second (J/s).
  - Joule is a unit of energy. However, the unit of energy that is used in electrical and computer engineering is a kilowatt-hour.
    - This is the unit used by the electric company when they read your electric meter to determine how much energy you have consumed each month.
  - Horsepower (hp) is used when describing the energy generated by motors
    - 1 kW-hr = 3.6 MJ = 1.34 hp
The change in energy as a function of time is power, which is measured in watts (W).

\[ p = \frac{dw}{dt} = \frac{dq}{dq} \frac{dv}{dt} = vi \]
Power Dissipated by a Resistor

\[ p = iv = i(iR) = i^2R \]

\[ p = iv = (v/R)v = v^2/R \]

\[ p = iv = i(i/G) = i^2/G \]

\[ p = iv = (vG)v = v^2G \]
Positive vs. Negative Power

- Power consumed/dissipated by a component is positive power

\[ P = +1W \]
Passive Sign Convention

- Generated power has a negative sign
  \[ P = -1W \]
Power (con’t)

• Since R and G are always real positive numbers
  – Power dissipated by a resistor is always positive
• The power consumed by the resistor is not linear with respect to either the current flowing through the resistor or the voltage dropped across the resistor
  – This power is released as heat. Thus, resistors get hot as they absorb power (or dissipate power) from the circuit.
Short and Open Circuits

- There is no power dissipated in a short circuit.

\[ P_{sc} = v^2 R = (0V)^2 (0\Omega) = 0W \]

- There is no power dissipated in an open circuit.

\[ P_{oc} = i^2 / R = (0A)^2 / (\infty \Omega) = 0W \]
Example 1

1V is dropped across some element (in red) and the wires to that element are connected directly to the independent current source.
Example 1

This means that 1V is also dropped across the independent current source. Therefore, the current source is generated $1 \text{ V}(3 \text{ A}) = 3 \text{ W}$ of power.

Passive sign convention: When current leaves the + side of a voltage drop across the independent current source, the power associated with the current source is:

$$p = -3 \text{ A}(1 \text{ V}) = -3 \text{ W}$$
Example 1

Conservation of energy means that the other element in red must be dissipating 3 W of power.

\[
\sum p = p_{\text{current source}} + p_{\text{red element}} = 0
\]

\[p_{\text{current source}} = -3 \text{ W}; \text{ therefore, } p_{\text{red element}} = 3 \text{ W}\]

Passive sign convention: When current enters the + side of a voltage drop across the element in red, the power associated with this element is:

\[p = 3 \text{ A}(1 \text{ V}) = 3 \text{ W}\]
Example 1

Suppose the red element was an independent voltage source. This means that the independent current source happens to be supplying power to the independent voltage source, which is dissipating power. This happens when you are charging a battery, which is considered to be an independent voltage source.
Power Supplies and Batteries

- Regulated power supply means that there is internal circuitry that maintains the output voltage and current to the values that you set.
  - There are some changes in the voltage from the wall plug (line voltage).
- The output from an unregulated power supply may fluctuate with changes in line voltage, changes in the resistors in the circuit (the load), temperature, etc.
Efficiency ($\eta$)

- The ratio of the power delivered to a load to the power delivered to the circuit.
  \[
  \eta = \frac{P_{out}}{P_{in}}
  \]

- Power loss is the internal power dissipated.
  \[
  P_{in} = P_{out} - P_{loss}
  \]

- Inexpensive power supplies can have an efficiency of less than 25%.
Batteries

• Specifications on batteries are:
  • The nominal voltage that it will generate.
    • Usually through an electrochemical reaction.
  • Ampere-hours or amp-hours
    • Determines how long a battery can deliver a certain amount of current at the nominal voltage.
  • Shelf life
  • Cycle time and Maximum Cycles
    • For rechargeable batteries.
  • Operating and storage temperatures
    • The electrochemical reaction typically is exothermic so the max. operating temperature is lower than the max. storage temperature.
Amp-Hours

- Supposed you purchased an trolling motor and battery for a boat.
  - The trolling motor is a 5hp 12V motor with a maximum rating of 30A
  - The battery is a 12V and can deliver 75 amp-hours.
  - So, how long can you go out on the water before the battery is dead?

\[ t = \frac{75A - hr}{30A} = 2.5hr \]
Summary

- Energy equal to the integration of power with respect to time.
- The power dissipated by a resistor can be written as:
  \[ p = \frac{v^2}{R} \]
  \[ p = i^2 R \]
  \[ p = v^2 G \]
  \[ p = i^2 / G \]

- The power dissipated by a resistor, which will be a positive number, is not linear with either the voltage across the resistor or the current flowing through the resistor.