MatLAB

Transfer Functions,
Bode Plots, and
Complex Numbers
Defining a Vector

Three elements
t=[0,.1,3] or t=[0 .1 3]
t = 0 0.1000 3.0000

Four Elements
t=[0:3]
t = 0 1 2 3

Elements that include 0, then 0.1, and then increments of 1 until it reaches the largest number ≤ 3
t=[0,.1:3]
t = 0 0.1000 1.1000 2.1000

Thirty one elements between 0 and 3 in increments of 0.1
t=[0:.1:3]
t = 0 0.1000 0.2000 0.3000 0.4000 0.5000 0.6000 0.7000 0.8000 0.9000 1.0000 1.1000 1.2000 1.3000 1.4000 1.5000 1.6000 1.7000 1.8000 1.9000 2.0000 2.1000 2.2000 2.3000 2.4000 2.5000 2.6000 2.7000 2.8000 2.9000 3.0000
Suppose you have determined that the transfer function for your filter is:

\[ H(\omega) = \frac{V_o}{V_{in}} = \frac{A_n s^n + A_{n-1} s^{n-1} + \cdots + A_1 s + A_0}{B_m s^m + B_{m-1} s^{m-1} + \cdots + B_1 s + B_0} \]

- You can put this transfer function into MatLAB
  - Define two vectors
    - \( A = [A_n, A_{n-1}, \ldots, A_1, A_0] \) where \( n, m \geq 0 \) and
    - \( B = [B_m, B_{m-1}, \ldots, B_1, B_0] \) \( n \) does not have to equal \( m \)
  - \( H = \text{tf}(A,B) \)
Example

High pass filter with a single-pole/single-zero

\[ H(\omega) = \frac{V_o}{V_{in}} = \frac{sRC}{1 + sRC} \]

• Let \( RC = 10^4 \) s/rad
  - \( A = [0, 10e3] \)
  - \( B = [1 \ 10e3] \)
  - \( H = \text{tf}(A,B) \)

\[
H = \frac{10000s}{10000s + 1}
\]
is returned when you run the program
MatLAB: Bode Plots

• Once you have entered the transfer function into MatLAB, you can use a predefined function ‘bode’ to automatically generate plots of the magnitude and phase vs. frequency.

Enter: bode(H)
Bode Plot Parameters

Control System Toolbox™

bode

Bode diagram of frequency response

Syntax

- \texttt{bode(sys)}
- \texttt{bode(sys,w)}
- \texttt{bode(sys1,sys2,...,sysN)}
- \texttt{bode(sys1,sys2,...,sysN,\textit{PlotStyle1},...,sysN,\textit{PlotStyleN})}

Description

bode computes the magnitude and phase of the frequency response of LTI models. When you invoke this function without left-side arguments, \texttt{bode} produces a Bode plot on the screen. The magnitude is plotted in decibels (dB), and the phase in degrees. The decibel calculation for \texttt{mag} is computed as \(20 \log_{10} |H(j \omega)|\), where \(|H(j \omega)|\) is the system's frequency response. You can use \texttt{bode} plots to analyze system properties such as the gain margin, phase margin, DC gain, bandwidth, disturbance rejection, and stability.

\texttt{bode(sys)} plots the Bode response of an arbitrary LTI model \texttt{sys}. This model can be continuous or discrete, and SISO or MIMO. In the MIMO case, \texttt{bode} produces an array of Bode plots, each plot showing the Bode response of one particular I/O channel. The frequency range is determined automatically based on the system poles and zeros.

\texttt{bode(sys,w)} explicitly specifies the frequency range or frequency points for the plot. To focus on a particular frequency interval \([\texttt{wmin}, \texttt{wmax}]\), set \(w = [\texttt{wmin}, \texttt{wmax}]\). To use particular frequency points, set \(w\) to the vector of desired frequencies. Use \texttt{logspace} to generate logarithmically spaced frequency vectors. Specify all frequencies in radians per second (rad/s).

\texttt{bode(sys1,sys2,...,sysN)} or \texttt{bode(sys1,sys2,...,sysN,w)} plots the Bode responses of several LTI models on a single figure. All systems must have the same number of inputs and outputs, but they can include both continuous and discrete systems. Use this syntax to compare the Bode responses of multiple systems.
Complex Numbers

• The default symbol for $\sqrt{-1}$ is $i$. However, MatLAB does recognize that $j$ is equivalent to $i$.
  – The coefficient of the imaginary number must be placed before the ‘$i$’ or ‘$j$’.
    • If you typed $c = 2-3j$, MatLAB interprets it as $c = 2.0000 - 3.0000i$
  – To find components of complex number
    • $\text{real}(c)$ returns 2
    • $\text{imag}(c)$ returns -3
Magnitude and Phase

• Magnitude of a complex number
  – Is the square root of the square of the real component plus the square of the imaginary component

• Phase of a complex number
  – Is the arc tangent of the imaginary component divided by the real component
    • Must change to degrees if the output of the arc tangent is given in radians