As will be seen in ECE 3004/3074 for those of you who are electrical engineering majors, RLC networks are commonly used as bandpass and notch filters to selectively remove certain frequencies from the signal delivered to a load, in oscillators, and in circuits that used in energy storage and power conversion such as pulse discharge circuits and voltage multipliers. As mentioned in the lecture, there are three different transient responses in RLC circuits – overdamped, critically damped, and underdamped. Engineers select the type of damping when designing circuits to achieve a certain performance.

In the case of the pulse discharge circuit, an overdamped response is preferred as all of the energy stored in the inductor or capacitor is quickly dissipated by the resistor in the RLC network. The capacitor in a series RLC network and the inductor in the parallel RLC network determine the amount of stored energy. The objective in a pulse discharge circuit is to design an RLC network that releases most of the stored energy through the resistor within a specified amount of time, usually as short as possible. Thus, an engineer must consider both the natural frequency \( \omega_0 \) and the damping factor \( \alpha \) when selecting the type of damping.

The transition between initial and final conditions for component voltages and currents is fastest in a critically damped circuit. You can always design a critically damped circuit, but to have an RLC network that operate always at the point where \( \omega_0 = \alpha \), requires components that (a) have extremely tight tolerances or are tunable (e.g., a trim potentiometer and variable capacitor) so that the magnitudes of the components can be tweaked to the required values, (b) that have very little dependence on environmental variables such as ambient temperature and (c) that behave as ideal component over the current, voltage, and frequency ranges of operation for the life of the circuit. There are parasitic resistances, capacitances, and inductances in a number of components – even a simple resistor such as any of the ones in your parts kit acts like an inductor in series with the parallel combination of a capacitor and the resistor. Hence, an RLC network that operates as a critically damped circuit is difficult to achieve.

Although essential in certain oscillator circuits, the underdamped response can be undesirable as it can take a long time for the signal to reach steady-state. The signal in an underdamped RLC network will overshoot – the signal will temporarily change sign as the signal decays to zero in the case of a natural response or it can reach a much larger magnitude than the magnitude of the final steady-state signal in the case of a forced response. The overshoot, which is also known as ringing, oscillates at the damped frequency of the RLC network \( \omega_d = \sqrt{\omega_0^2 - \alpha^2} \) and the rate of the exponential decay of
the oscillations is determined by the damping factor. The magnitude of the overshoot must be taken into consideration when determining the maximum voltage and current specifications for the components selected for use in the circuit. In addition to stressing components electrically, the overshoot of an underdamped circuit can also stress components mechanically because of rapid thermal expansion caused by a spike in the component’s temperature as a result of the dissipated power. The stress can damage components, enhancing their nonideal properties, and can reduce the components’ useable lifetime.

In this experiment, we will explore how the value of $\alpha$ affects the ringing that occurs in an underdamped series RLC network. The circuit that will be analyzed is shown in Figure 2. $\alpha$ will be altered by varying the value of $R_1$. Note that this circuit is a bit more complicated than a simple series RLC circuit. First, there is a Thévenin equivalent resistor $R_{th}$, which is in series with the voltage source $V_1$. This resistor is used to model the resistance of the arbitrary waveform generator, which is a nonideal voltage source. Next, there is a resistor $R_2$ that is in parallel with the RLC network. This resistor is used to insure that the energy stored in the capacitor does not discharge directly into the arbitrary waveform generator, which could cause the current to flow into the voltage source. Depending on the voltage source used, this may cause the voltage source to turn off and could possibly damage the arbitrary waveform generator. Instead, the transient current generated while the capacitor discharges will be added to a dc current $V_1/(R_{th} + R_2)$. The values that will be used in this experiment for $V_1$, the inductor, the capacitor, and the resistors have been selected to insure that the arbitrary function generator will always act as a power generating element in the circuit. Lastly, a lumped component model for a nonideal inductor is used – a series combination of an ideal inductor $L_1$ and a resistor $R_{parasitic}$, which is the resistance of the wire used to create the inductor.

![Fig. 2: Schematic of a series RLC circuit.](image-url)

NOTE: Caution should be used when designing a RLC network to insure that the voltage and current specifications for the voltage source are not exceeded or it may be damaged.
Analysis:

1. Determine the natural frequency, damping factor, and damping frequency of the circuit shown in Figure 2 where $R_{th} = 50\,\Omega$, $R_{parasitic} = 10\,\Omega$ and a) $R_1 = 56\,\Omega$ and b) $R_1 = 1k\,\Omega$. Note that you will have to combine $R_{th}$, $R_{parasitic}$, $R_1$, and $R_2$ into one equivalent resistor before performing these calculations.

2. For the two cases a) $R_1 = 56\,\Omega$ and b) $R_1 = 1k\,\Omega$, determine the values for $A_1$ and $A_2$ in the equation for a forced response, $v_c(t) = [A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)]e^{-\alpha t} + V_0$ when $V_1 = 2V[1 + u(t)]$. Note that $V_0 \neq V_1$. The solution will be the same as the case where $V_1 = 2V u(t)$ because the transient response is only produced by the step function, not the d.c. component of $V_1$.

3. Using MATLAB, plot the equations for the voltage across the capacitor developed in Step 2 on the same graph from $t = 0\,\text{s}$ to $t = 0.2\,\text{ms}$.

Modeling

4. In PSpice, simulate the transient response of the circuit shown in Figure 2 where $R_{th} = 50\,\Omega$, $R_{parasitic} = 10\,\Omega$ when a) $R_1 = 56\,\Omega$ and b) $R_1 = 1k\,\Omega$. Set the amplitude of the voltage source such that the output of $V_1$ is a 2 V square wave with a 2 V d.c. offset. The pulse width should be 0.2 ms and the period should be 0.4 ms.

5. Graph the voltage across $C_1$, the voltage across $R_1$, and the voltage across $R_2$ for one period of the square wave. Either scale the curves so that the magnitude and shape of each signal is readable or add plots to obtain a stack of graphs of the signals.

6. Graph the current flowing out of $V_1$ to show that the voltage source is always a power generating element in the circuit.

7. Measure the period of the underdamped response of the voltage across the capacitor when a) $R_1 = 56\,\Omega$ and b) $R_1 = 1k\,\Omega$. Calculate the damped frequency $\omega_d$ using the period measured. Determine the percent difference between the expected value of $\omega_d$ calculated in Step 1 and the damped frequency calculated from the data graphed in PSpice.

Measurements

8. Measure the parasitic resistance $R_{parasitic}$ of a 1 mH inductor using your DMM.

9. Construct a modification of the circuit shown in Figure 2. These modifications are as follows:
   a. If you are using the Velleman PCSGU250, do not include $R_{th}$ as the arbitrary function generator has a Thévenin equivalent resistance of 50 $\Omega$.
   b. If you are using the Digilent Analog Discovery, you must include a 50 $\Omega$ resistor in series with the output of the arbitrary function generator. To do this, you may set the jumper to 50 $\Omega$ on the BNC adapter board (Fig. 3) or you should use a 50 $\Omega$ resistor in the circuit as $R_{th}$.
   c. Do not include a 10 $\Omega$ resistor for $R_{parasitic}$ as this resistor is part of your (nonideal) inductor.
10. Select the square wave with a 50% duty cycle on the arbitrary waveform generator. Set the frequency so that the period of the square wave is 0.4 ms. The amplitude should be equal to 2 V and the d.c. offset should be 2 V.

11. Measure the voltage across $C_1$ and the voltage across $R_1$ when $R_1 = 56 \, \Omega$ for one period. Determine the damped frequency $\omega_d$ of the transient response when the input voltage increases from 2 V to 4 V and, again, when the input voltage decreases from 4 V to 2 V. Calculate the percent difference between the damped frequency found in Step 1 and the measured frequency.

12. Measure the voltage across $C_1$ and the voltage across $R_1$ when $R_1 = 1k \, \Omega$ for one period. Determine the damped frequency $\omega_d$ of the transient response when the input voltage increases from 2 V to 4 V and, again, when the input voltage decreases from 4 V to 2 V. Calculate the percent difference between the damped frequency found in Step 1 and the measured frequency.

Conclusion:

13. To reduce the ringing on the voltage across $R_1$:
   a. Should $\alpha$ increase or decrease?
   b. If $R_1$ is held constant, should value of $L_1$ increase or decrease to reduce the transient response?
   c. Reconsider your answer in part b when a real inductor is used when $R_1$ is small (i.e., 56 $\Omega$).

14. Given the relationships between energy, power, voltage, and resistance, why would you expect that the energy stored in the capacitor will dissipate faster (i.e., the ringing on the voltage across the resistor or capacitor) as $R_1$ increases?

15. Given the relationship between energy, charge, voltage, and capacitance, would you expect that the amplitude of the voltage overshoot in the transient response to increase or decrease if the capacitance of $C_1$ is increased while the magnitudes of the other components in the circuit are held constant?