A Differentiator Circuit
All of the diagrams use a uA741 op amp.
  ◦ You are to construct your circuits using an LM 356 op amp.

There is a statement that values for \( R_1 \), \( R_2 \), and \( C_2 \) should be limited by what is in your parts kit.
  ◦ You should combine multiple resistors (capacitors) to obtain a desired resistance (or capacitance).
    • You do not have to get the exact value. Come close – preferably within the tolerance of the resistors (capacitors) in the parts kit.
Figure 1: An ideal differentiating op amp circuit.
Capacitors

\[ i_C(t) = C \frac{dv_C}{dt} \]

\[ v_C(t) = \frac{1}{C} \int_{t_o}^{t_1} i_C(t) \, dt + v_C(t_o) \]
\[ i_R + i_C + i = 0 \]

where \( i = 0 \text{mA} \)

\[ i_C = C_1 \frac{dV_3}{dt} \]

\[ i_R = \frac{[0V - V_o]}{R_1} \]

\[ V_o = -R_1C_1 \frac{dV_3}{dt} \]

*Figure 1: An ideal differentiating op amp circuit.*
Figure 2: A practical differentiator circuit.
Why The Second Circuit?

- If the input signal contains electronic noise with high frequency components, the magnitude of the high frequency components will be amplified significantly over the signal of interest (look at the equation for the output voltage after you have taken the derivative of the input voltage).
  - The circuit may become unstable – and certainly the shape of the output signal will not be what you expected.

- It is necessary to modify the circuit to include a low pass filter to reduce or eliminate this effect.
There are two modifications to the circuit – both of which results in the formation of frequency filters.

- First, a series resistor $R_2$ is inserted before the negative input terminal of op amp. The effect of this resistor is to act as an attenuator for the high frequency components.
- Second, a capacitor $C_2$ is placed in the feedback network.
  - Because a capacitor acts like an open circuit at low frequencies and as a short circuit at high frequencies, $C_2$ doesn’t impact the operation of the circuit at low frequencies.
  - It acts as a short in parallel with $R_1$ in the feedback path for the high frequency components of the input signal. This means that the high frequency components of the output voltage will be equal to the voltage at the positive input terminal of the op amp (0 V).
Figure 2: A practical differentiator circuit.
Mathematically

\[ i_C(t) = C \frac{dv_C}{dt} \]

\[ v_C(t) = \frac{1}{C} \int_{t_o}^{t_1} i_C(t) dt + v_C(t_o) \]

If \( v_C(t) = a_v \sin(\omega t) \), then

\[ i_C(t) = -\omega a_v C \cos(\omega t) \]

\( i_C(t) \to 0A \) as \( \omega \to 0 \text{ rad/ s} \)

If \( i_C(t) = a_I \sin(\omega t) \), then

\[ v_C(t) = \frac{a_I}{\omega C} \cos(\omega t) \]

since \( v_o(t) = -v_C(t) \),

\( v_o(t) \to 0V \) as \( \omega \to \infty \text{ rad/ s} \)

At low frequencies

At high frequencies
When the voltage across the capacitor doesn’t change (i.e., d.c. voltage), the capacitor’s current is equal to zero.
- The capacitor acts like an open circuit.

When the voltage across the capacitor is changing rapidly (e.g., high frequency sine wave), the capacitor’s current is large and also changes with time.
- The capacitor acts like a short circuit. The current through the circuit is limited by the other components in the circuit (i.e., the resistors).

From these equations:
R2 with C1 forms a high pass filter.

If V3 is a d.c. voltage source, C1 acts like an open circuit and all of the input voltage (V3) is dropped across the capacitor ($V_{C1}$) and the current through R2 and C1 will be determined primarily by the first derivative of the V3.

If V3 is a high frequency a.c. voltage source, C1 acts like a short circuit and the current through R2 and C1 will be determined primarily by V3 divided by R2.
R1 with C2 forms a low pass filter.

If the difference in the voltage between the negative input terminal on the op amp and Vo is relatively constant, C1 acts like an open circuit and all of the current through R2 and C1 will flow through R1.

If the difference in the voltage between the negative input terminal on the op amp and Vo varies a lot with time, C1 acts like a short circuit and all of the current through R2 and C1 will flow through C2 and the output voltage will be approximately equal to the voltage on the negative input terminal, which will be 0 V.
$f_{\text{unity}}$ is the frequency where the gain of the amplifier is equal to 1.
Design Constraints

\[ f_C = \frac{1}{2\pi R_2 C_1} \text{ and } f_H = \frac{1}{2\pi R_1 C_2} \]

\[ f_{\text{unity}} = \frac{1}{2\pi R_1 C_1} \]

\[ \frac{1}{R_1 C_1} \leq \omega \leq \frac{1}{R_2 C_1} < \frac{1}{R_1 C_2} \]

\[ \omega = 2\pi f \]
Design Constraints

\[ f_C = 3 \text{ kHz} \]
\[ f_H = 5 \text{ kHz} \]
\[ f_{unity} = 1.5 \text{ kHz} \]
\[ C_1 = 0.1 \mu F \]
dB

- dB is an abbreviation for decibels

\[
dB = 10 \log\left(\frac{P_{out}}{P_{in}}\right)
\]

- 3dB occurs when \(\frac{P_{out}}{P_{in}} = \frac{1}{2}\)

\[
dB = 20 \log\left(\frac{V_{out}}{V_{in}}\right)
\]

- 3dB occurs when \(\frac{V_{out}}{V_{in}} = \frac{\sqrt{2}}{2} = 0.707\)
Follow the Directions in the Lab Manual

Except:

- Use the function generator on the Velleman oscilloscope
  - Remember that you have to set the Amplitude to 10V to have $5V \sin(\omega t)$ outputted.
- Do not use the 10X probes with the Velleman oscilloscope when performing the oscilloscope measurements.
  - Just use the standard BNC-to-alligator or BNC-to-IC clip cables.
- All plots aside from the ones generated using PSpice should be made using MatLAB.
PSpice Simulation: AC Sweep

![Diagram showing differentiator, integrator, and gain](chart.png)
Phase Shift

\[
\frac{d \sin(\omega t)}{dt} = \omega \cos(\omega t) = \omega \sin(\omega t + 90^\circ) = \omega \sin(\omega t + \phi)
\]

where \( \phi \) is the phase angle.

\[
\phi = -360 \frac{\Delta t}{T} \text{ degrees}
\]

where the period of the sine wave is \( T = \frac{1}{f} = \frac{2\pi}{\omega} \).
The phase shift between the input voltage and the output voltage of the op amp will change from $90^\circ$ to $180^\circ$ to $270^\circ$ as the operation of the circuit changes from a differentiator to inverting amplifier to integrator.
Information in first half cycle is incorrect because PSpice sets the initial charge on the capacitor at 0 V.
Measurement of Phase Angle

- There are two sets of instructions in the Week 9 folder under Modules.
  - Phase Delay, which explains how to make a phase angle calculation using the information displayed when the Oscilloscope function of the Velleman oscilloscope is used.
    - You should become familiar with this technique.
  - Magnitude and Phase, which explains how to use the automated measurement tools on the Velleman scope to obtain the magnitude and phase of a signal at a single frequency and over a range of frequencies.