Solutions to Example Exam 3
Solutions to Sample Exam 3

1. A 1st order circuit is an RC or RL circuit. After a disturbance to the magnitude of the power supply in the circuit, it will be 5% before the circuit reaches steady state.
2. There are several different way to write the equation that describes the voltage shown in the graph. If you assume that the voltage is equal to 3V between \(-\infty < t < 5\) s.

\[ V(t) = 3V - 1V \ u(t-5s) - 0.4V/s \ r(t-10s) + 0.4V/s \ r(t-20s) + 2V \ u(t-20s) \]
3. Over damped
   Critically damped
   Under damped

Shapes of responses as power source in circuit turns off. The change with respect to time is more rapid initially for overdamped response, but critically damped response reaches zero sooner.
4. A forced response is one in which a voltage or current source in the circuit is initially zero and then, at some time \( t_0 \), turns on.
5. First, figure out what the voltage and current sources are at \( t = -10 \text{s} \).

\[ 3A \ u(15\text{s} - (-10\text{s})) = 3A \ u(25\text{s}) \]

This current source is on since the argument within the brackets for the unit step function is greater than zero.

\[ 15V \ u((-10\text{s}) + 5\text{s}) = 15V \ u(15\text{s}) \]

The voltage source is also on.
In steady state, inductors are short circuits and capacitors are open circuits.
6. a. Inductors are short circuits in steady state.

The voltage source turns on at $t = -5s$.

Circuit to use when solving for initial conditions.
Circuit to use when solving for final conditions.

\[ V_L\ \text{initial} = 0V \quad \text{as} \quad V_L = i_L(0^+) = 0V \]

\[ V_L\ \text{final} = 0V \]
$I_{\text{initial}} = 0 \text{mA}$ since there are no voltage or current sources in the initial circuit.

$$I_{L, \text{final}} = \frac{V_2}{R_1} = \left[\left(\frac{R_3}{R_3 + R_2}\right) 6 \text{V}\right] / R_1 = 1 \text{mA}$$

C. Forced response since the voltage source turns on at $t = -5 \text{s}$. 

d. $Z = \frac{L}{R} = \frac{50 \mu \text{H}}{2k\Omega} = 25 \text{ns}$

e. $I_L = I_{L, \text{final}} + (I_{L, \text{initial}} - I_{L, \text{final}}) e^{-(t-t_0)/Z}$ for $t > t_0$. 
\[ i_L(t) = \text{mA} \left[ 1 - e^{-\frac{(t+t_0)}{2\text{ns}}} \right] \]

\[ i_L = i_{R_1} = i_f = \text{mA} \left[ 1 - e^{-\frac{(t+t_0)}{2\text{ns}}} \right] \]

\[ V_0 = v_f R_f + u_2 = 38V \left[ 1 - e^{-\frac{(t+t_0)}{2\text{ns}}} \right] + 2V \]
7. a. \( \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{((0.3 \mu F)40 mH)^{1/2}} = 9.13 \text{krad/} s \)

b. \( \alpha = \frac{1}{2RC} \) for parallel RLC circuit

\[
\begin{align*}
\text{Parallel RLC Circuit} & \\
\text{2SV}[1-u(t)] & \\
\text{0.3} \mu \text{F} & \\
R & \\
40 mH &
\end{align*}
\]
For critically damped circuit

\[ \alpha = \omega_0 = \frac{9.13 \text{ krad}}{5} \]

\[ R = \left\{ \frac{9.13 \text{ krad}}{5} \left[ 2 \left( 0.3 \mu\text{F} \right) \right] \right\}^{-1} = 183 \Omega \]

c. The voltage source is equal to 25V when \( t < 0 \) s and is equal to 0V when \( t > 0 \) s.
In steady state

\[ i_L = 0.137A, \quad i_R = 0A, \quad i_C = 0A, \quad v = 0V \]

Final condition

\[ i_L = i_R = i_C = 0A, \quad v = 0V \]
\[ V_c(t) = V_L(t) = L \frac{d}{dt} i_L(t) \]

\[ i_L(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t} \]

\[ V_c(t) = -L\alpha A_1 e^{-\alpha t} - LA_2 \alpha t e^{-\alpha t} + LA_2 e^{-\alpha t} \]

Find \( A_1 \) and \( A_2 \)

\[ i_L(0^+) = A_1 = 0.137 A \]

\[ V_L(0^+) = 0 = -L\alpha \left[ A_1 + A_2 t - \frac{1}{\alpha} A_2 \right] \]

\[ 0 = 0.137 A - \frac{1}{9.13 \text{broad}} A_2 \quad A_2 = 1.2 S k A \]
Series RLC circuit so

\[ \alpha = \frac{R}{2L} = 2 \omega_0 \quad \text{where} \quad \omega_0 = \frac{1}{\sqrt{3mH(5\mu F)}} \]
\( \omega_0 = 8.16 \text{ krad/s} \)

\( \alpha = 2\omega_0 = 16.3 \text{ krad/s} = \frac{R}{2L} \quad R = \)

b. Since the switch closes at \( t = 5s \)

\( t < 5s \)
\( i_L = i_c = i_R = 0A \quad V_L = V_c = V_R = 0V \)

\( t > 5s \)
\( i_L = i_c = i_R = 0A \quad V_L = V_R = 0V \quad V_c = 4A(R) \quad \mathcal{U}_e = 392V \)
\[ U_c(t) = A_1 e^{s_1(t - s_s)} + A_2 e^{s_2(t - s_s)} + U_{\text{final}} \]

\[ s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2.21 \text{krad/s} \]

\[ s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -30.11 \text{krad/s} \]

To find \( A_1 \) and \( A_2 \)

\[ U_c(s_s) = OV = A_1 + A_2 + 392 \text{V} \]

\[ i_c(s_s) = OA = C \frac{d}{dt} U_c(t) \]

\[ 0 = A_1 s_1 + A_2 s_2 \]
\[ A_2 = -\frac{s_1}{s_2} A_1 \]

\[ A_1 = \frac{392V}{1 - \frac{s_1}{s_2}} = 423V \]

\[ A_2 = -31.1V \]

\[ \dot{u}_c(t) = C \left[ s_1 A_1 e^{s_1(t-t_0)} + s_2 A_2 e^{s_2(t-t_0)} \right] \]