Solutions to Exam 1
Problem 1

\[ q = \left[ 3 + 2t + 0.5 \sin(\omega t) - 0.1e^{-5t} \right] C \]

To find current, \( i \)

\[ i = \frac{dq}{dt} \]

\[ i = \left[ 0 + 2 + 0.5 \omega \cos(\omega t) + 0.5e^{-5t} \right] A \]
Problem 2a

- Passive sign convention must be used.

\[ p = vi \]

Any component
\[ P = + (V)(I) \]

Any component
\[ P = - (V)(I) \]
Note if the current and/or voltage has a sign along w/ the numerical value, you use it.

\[ P = + (2v)(-A) = -2\text{W} \]

See left hand set of circuits on previous page.
Problem 2a (con’t)

Because #3 and #4 are in parallel

#1 \[ P = +3V \times 1A \]
\[ = +3W \]

#2 \[ P = +2V \times 1A \]
\[ = +2W \]

#3 \[ P = -(5V \times 3A) \]
\[ = -15W \]

#4 \[ P = +(5V \times 2A) \]
\[ = +10W \]
Since the power is positive for components #1, #2, and #4, these components dissipate power. Since the power is negative, component #3 generates power or supplies the power to the circuit that the other components dissipate.
Problem 2b

• From the conservation of energy, the sum of the power in a circuit must be equal to zero.
  – If the resistors in parallel are replaced by an equivalent resistor, then the same current flows through all of the components.
  • Substituting in the equation for power, we find that the sum of all voltages in a loop must be equal to zero.
Problem 2b (con’t)

When arrow enters - side of voltage drop, we subtract it from the total. Going clockwise, we next come to the + side of the voltage on #2, which will be added.

\[-3V + 2V + 6V - V_x = 0\]

\[V_x = 5V\]
Component #1 is in series with the unlabeled component. All of the current must flow from one component into the other.

However, the direction of $I_x$, indicated by the arrowhead, is in the opposite direction to the arrowhead of the 4A current. To account for this: $I_x = -4A$. 
Problem 3

a. A ¼ watt 100Ω resistor will be used in a circuit. Based on this power limitation:
   
i. What is the maximum current that should flow through this resistor?

   ii. What is the maximum voltage that should be applied across this resistor?

   iii. If two ¼ watt 50Ω resistors were used instead of the ¼ watt 100Ω resistor, does the maximum current increase, decrease, or remain the same? (No calculations are necessary.)
Problem 3 (con’t)

a. \( P = 0.25\, \text{W} \)

\( R = 100\, \Omega \)

\[ P = I^2 R \]

\[ 0.25\, \text{W} = I^2 (100\, \Omega) \]

\[ I = 5 \times 10^{-2} \, \text{A} = 50 \, \text{mA} \]

This is the current that will cause the 100\, \Omega resistor to dissipate 0.25\, \text{W}. Any more current and the resistor will be damaged.
Problem 3 (con’t)

b. $P = 0.25 \, \omega$

$R = 100 \pi$

$P = \frac{V^2}{R}$

$V = \sqrt{PR} = SV$
Problem 3 (con’t)

\[ P = I^2 R \]

\[ I_{\text{max}} = 50 \text{ mA} \]

\[ P = I^2 R \text{ for each } 50\Omega \text{ resistor} \]

\[ 0.25 \text{ W} = I^2 (50\Omega) \]

\[ I_{\text{max}} = 70.7 \text{ mA} \]

2 ¼ W 50 Ω in series can handle more current than the ¼ W 100 Ω resistor.
I work from the furthest resistor towards the 2 terminals

\[ R_{eq1} = \frac{4\Omega \cdot 12\Omega}{4\Omega + 12\Omega} = 3\Omega \]
Problem 4 (con’t)

\[ R_{eq1} = \frac{1R + 3R}{4} = 4R \]

\[ R_{eq2} = \frac{10R + 5R}{4} = 4R \]
Problem 4 (con’t)

\[ \frac{4\Omega}{4\Omega} \implies Req_3 = \frac{4\Omega(4\Omega)}{4\Omega+4\Omega} = 2\Omega \]

\[ Req_3 \implies Req_4 = 4\Omega + 2\Omega = 6\Omega \]
Problem 4 (cont'd)

\[ \text{Req} = 10 \Omega \]
\[ \text{Req} = 5 \Omega \]
\[ \text{Req} = 3 \Omega \]
\[ \text{Req} = 10 \Omega \]

\[ \text{Req}_s = \frac{3n \cdot 6n}{3n + 6n} = 2n \]

\[ \text{Req} = 2n \]

\[ \text{Req} = 10n + 5n + 2n = 17 \Omega \]
Problem 5a

\[ R_{eq} = \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right]^{-1} \]
\[ R_{eq} = \left[ \frac{1}{3k\Omega} + \frac{1}{3k\Omega} + \frac{1}{3k\Omega} \right]^{-1} \]
\[ R_{eq} = 1k\Omega \]
\[ I_T = \frac{9V}{3\Omega} = 3mA \]

Current Division

\[ I = \frac{R_{eq}}{R_3} I_T \]

\[ I = \frac{1\Omega}{3\Omega} (3mA) \]

\[ I = 1mA \]
Problem 5b

\[ R_{eq} = \frac{2k\Omega \times 2k\Omega}{2k\Omega + 2k\Omega} = 1k\Omega \]
Voltage Division

\[ V = \frac{R_1}{R_1 + R_{eq}} \quad V_T = \frac{2kn}{2kn + 11cn} \quad (6V) \]

\[ V = 4V \]
Problem 6

- We haven’t discussed PSpice in any reasonably manner so nothing related to this program will show up on the exam.
Problem 7

Nodal Analysis

Step 1: Pick a node to be the reference node (ground). Any node is acceptable.
Problem 7 (con’t)

Label nodes and draw currents through components with direction of flow.
Problem 7 (con’t)

KCL
Node A: $I_1 = I_2 + I_3$
Node B: $I_3 = I_4 + 3mA$
Node C: $I_2 + I_4 = I_5$
Node D: $I_5 + 3mA = I_6$

Note that $I_1 = I_6$ because the 1kΩ resistor is in series with the 10V source.
Problem 7 (con’t)

Ohm’s Law
\[ I = \frac{[\text{voltage at node I leaving}] - [\text{voltage at node I going}]}{\text{divided by resistance}} \]

Can’t write an equation for \( I_1 \) as ideal voltage sources have no resistance.

\[ I_2 = \frac{(V_A - V_C)}{1k} \]
\[ I_3 = \frac{(V_A - V_B)}{5k} \]
\[ I_4 = \frac{(V_B - V_C)}{4k} \]
\[ I_5 = \frac{(V_C - V_D)}{2k} \]
\[ I_6 = \frac{V_D}{1k} \]
Problem 7 (cont')

Final set of equations: Substitute equations from Ohm's Law into equations from KCL.

Node A: \( \frac{V_D}{1k\Omega} = \frac{V_A-V_C}{1k\Omega} + \frac{V_A-V_B}{5k\Omega} \)

Node B: \( \frac{V_A-V_B}{5k\Omega} = \frac{V_B-V_C}{4k\Omega} + 3mA \)

Node C: \( \frac{V_B-V_C}{4k\Omega} + \frac{V_A-V_C}{1k\Omega} = \frac{V_C-V_D}{2k\Omega} \)

Node D: \( \frac{V_C-V_D}{2k\Omega} + 3mA = \frac{V_D}{1k\Omega} \)

This is when you stop in the exam.
Problem 8

Identify mesh loops (4 total).

Draw mesh currents.
Problem 8 (con’t)

Label voltage across components. Since one is a voltage source, it already has a known voltage so you can skip it.

You must have a voltage across the current source.
Problem 8 (con’t)

Use KVL.
Note that \( i_4 = -7 \text{mA} \)

Mesh #1: \[ V_2 + V_4 - V_3 = 0 \]
Mesh #2: \[ 3V - V_6 - V_4 = 0 \]
Mesh #3: \[ -V_1 + V_3 + V_5 = 0 \]
Mesh #4: \[ -V_5 + V_6 + V_7 = 0 \]
Problem 8 (con’t)

Use Ohm's Law.

You can not write a relationship for the voltage or current source.

\[
V_1 = -i_3(1\text{kn})
\]
\[
V_2 = i_1(3\text{kn})
\]
\[
V_3 = (i_3 - i_1)(1\text{kn})
\]

where \( i_4 = -7\text{mA} \)

\[
V_4 = (i_1 - i_2)(1\text{kn})
\]
\[
V_5 = (i_3 - i_4)(2\text{kn})
\]
\[
V_6 = (i_4 - i_2)(8\text{kn})
\]
Problem 8 (con’t)

Final set of equations: Substitute equations from Ohm’s Law into equations from KVL.

Mesh #1 \[ i_1 (3k\Omega) + (i_1 - i_2)(1k\Omega) - (i_3 - i_1)(1k\Omega) = 0 \]

Mesh #2 \[ 3V - (i_4 - i_2)(8k\Omega) - (i_1 - i_2)(1k\Omega) = 0 \]

Mesh #3 \[ -(-i_3)(1k\Omega) + (i_3 - i_1)(1k\Omega) + (i_3 + 7mA)(2k\Omega) = 0 \]

Mesh #4 \[ -(i_3 + 7mA)(2k\Omega) + (-7mA - i_2)(8k\Omega) + V_7 = 0 \]

This is where you can stop in the exam.