**Short and Open Circuits**

*Short Circuits:*

A short circuit can be considered to be a resistor, $R_{SC} = 0 \, \Omega$. If we apply Ohm’s Law to a short circuit, then:

$$V = I \times R_{SC} = I(0 \, \Omega) = 0 \, V$$

So, the electromotive force, $V$, required to move electrons through a short circuit as a function of time, $I$, is zero. Short circuits present a path for unlimited current to flow with no force required to push the current through the wire. Therefore, there is no restriction on the amount of current that can flow through a short circuit by the short circuit; the limitation is a result of the amount of current that the other components in the circuit allow to flow into or out of the short circuit.

Suppose you have a short circuit in parallel with a resistor, $R$, and you need to determine the amount of current that will flow through the resistor. The voltage across resistor $R$ will be equal to the voltage across the short circuit. We just found that the voltage across the short circuit is 0 V from Ohm’s Law. Therefore, the voltage across the resistor $R$ is 0 V.

If you calculate the equivalent resistance of a resistor $R$ in parallel with a short circuit is:

$$R_{eq} = \left(\frac{1}{R_{SC}} + \frac{1}{R}\right)^{-1} = \left[\left(\frac{1}{0 \, \Omega}\right) + \left(\frac{1}{R}\right)\right]^{-1} = \infty \, \Omega^{-1} + \left(\frac{1}{R}\right)^{-1}$$

Since infinity plus a number is still infinity:

$$R_{eq} = \infty \, \Omega^{-1} + \left(\frac{1}{R}\right)^{-1} = 0 \, \Omega$$

Or, if we use the equation specifically for two resistors in parallel:

$$R_{eq} = \frac{R_{SC} \times R}{R_{SC} + R} = \frac{0 \, \Omega \times R}{0 \, \Omega + R} = 0 \, \Omega$$

The voltage across the equivalent resistor is also equal to 0 V when you apply Ohm’s Law.

To calculate the current flowing through resistor $R$, apply Ohm’s Law again. $V_R = I_R (R)$ where $V_R = 0 \, V$. $I_R$ must be 0 A as long as $R > 0 \, \Omega$, independent of the magnitude of the total current, $I_T$, flowing into the parallel combination. Or, if we use the current division equation,

$$I_R = \left[\frac{R_{SC} \times R}{R_{SC} + R}\right] \times I_T = \left[\frac{0 \, \Omega}{0 \, \Omega + R}\right] \times I_T = 0 \, A$$

Conversely, the current flowing through the short circuit will be equal to the entire current flowing into the parallel combination of the short circuit and resistor $R$.

$$I_{SC} = \left[\frac{R}{R_{SC} + R}\right] \times I_T = \left[\frac{R}{0 \, \Omega + R}\right] \times I_T = I_T$$

Mathematically, two short circuits in parallel (i.e., $R_{SC1} = R_{SC2} = 0 \, \Omega$) will share the current $I_T$ equally.

$$I_{SC1} = \left[\frac{R_{SC2}}{R_{SC1} + R_{SC2}}\right] \times I_T = \left[\frac{0 \, \Omega}{0 \, \Omega + 0 \, \Omega}\right] \times I_T = 0.5 \times (0/0) \times I_T$$

Here, we can assume that $(0/0) = 1$ so $I_{SC1} = I_{SC2} = \frac{1}{2} \times I_T$.

If the short circuit is in series with the resistor $R$, then the equivalent resistance is equal to:

$$R_{eq} = R + R_{SC} = R + 0 \, \Omega = R$$

The voltage across the two resistors in series is equal to the voltage across the equivalent resistor. This means that the voltage across the resistor $R$ and the short circuit resistor is equal to the voltage across resistor $R$. The current through the resistor $R$ in series with the short circuit is solely determined by the magnitude of $R$.

$$I = \frac{V}{R_{eq}} = \frac{V}{R}$$

*Open Circuits:*

An open circuit can be considered to be a resistor $R_{oc} = \infty \, \Omega$. If we apply Ohm’s Law to an open circuit, then:

$$V_{oc} = I_{oc} \times R_{oc} = I_{oc} (\infty \, \Omega) = \infty \, V$$

So, the electromotive force, $V$, required to move electrons through a short circuit as a function of time, $I$, is infinite, no matter how small the current is. Since we do not have circuits that have an infinite amount of voltage available, the current that flows through an open circuit must be zero.
Note: In Electrical Engineering, infinity is defined as 1/0. This triggers an oddity about Ohm's Law. The fact that the current through an open circuit is always 0 A does not mean that the voltage across an open circuit is 0 V. The solution for the voltage across an open circuit:

\[ V_{oc} = 0A \left( \infty \Omega \right) = 0 \left( \frac{1}{0} \right) V \]

So, you might think, instead, that \( V_{oc} = 1V \) whenever you have an open circuit because the zeros in the fraction will cancel. However, the fraction 0/0 is not equal to 0 or 1. Nor is it equal to \( \infty \). If you think back to your calculus class where they discussed limits, limit of the product of 0 times \( \infty \) can be any value between 0 and \( \infty \), inclusive. From L'Hopital's Rule, the voltage across an open circuit will depend, mathematically, on the rate at which the resistance goes towards \( \infty \) \( \Omega \) compared to the rate at which the current is moving towards 0 A. Rather than apply L'Hopital's Rule to each open circuit to calculate the voltage across the open circuit, we apply Kirchhoff’s Voltage Law (KVL), which must hold even when there is voltage across the open circuit. KVL states that the sum of the voltages around a loop must be equal to 0 V. So, the voltage across an open circuit, \( V_{oc} \), is determined by the voltage rises and drops of the other components in the loop in which the open circuit exists.

If there is a resistor, \( R \), in parallel with the open circuit, the equivalent resistance of this combination is:

\[ R_{eq} = \left( \frac{1}{R_{oc}} + \frac{1}{R} \right)^{-1} = \left[ \frac{1}{0 \Omega} + \frac{1}{R} \right]^{-1} = \left[ \frac{1}{R} \right]^{-1} = R \]

To calculate the current flowing through resistor \( R \) using the current division equation,

\[ I_R = \frac{R_{oc}}{R_{oc} + R} I_T = \frac{\infty \Omega}{\infty \Omega + R} I_T \]

Again, since \( \infty + X = \infty \), \( I_R = I_T \).

Any real number divided by infinity is equal to 0 or \( R/(1/0) = R(0)/1 = 0 \). Therefore, no current flows through the open circuit.

The voltage across resistor \( R \) is equal to:

\[ V_R = I_R R \]

Since the open circuit is in parallel with \( R \),

\[ V_{oc} = V_R = I_T R \]

If resistor \( R \) is in series with the open circuit, then the equivalent resistance is:

\[ R_{eq} = R_{oc} + R = \infty \Omega \]

Therefore, the current through resistor \( R \) will be equal to 0 A because we can't have an infinite voltage drop across the two resistors in series. The voltage across resistor \( R \) must then be 0 V. So the total voltage across the two resistors in series, \( V_T \), is equal to the voltage drop across the open circuit resistor, \( V_{oc} \). If you use voltage division equation to prove this:

\[ V_R = \frac{R}{R + R_{oc}} V_T = \frac{R}{\infty \Omega + R} V_T = 0 \ V \]

\[ V_{oc} = \frac{R_{oc}}{R_{oc} + R} V_T = \frac{\infty \Omega}{\infty \Omega + R} V_T = V_T \]

\( V_T \) must be calculated using Kirchhoff’s Voltage Law or other technique and depends on the other components in the circuit.