Series RLC Network

An example on how to solve for $A_1$ and $A_2$
Series RLC Circuit

Note that these two circuits are the same.

A single pole/double throw switch (a switch that is composed of one wire that is moving between two points in the circuit) in the upper circuit causes the 5 V supply to be removed from the circuit at \( t = 0 \) s.

The step function in the lower circuit causes the output of the voltage supply to change from 5 V to 0 V at \( t = 0 \) s.
• The unit step function, $u(t)$, in this circuit has the following definition:

\[
\begin{align*}
u(t-t_0) \text{ where } t_0 &= 0s \\
u(t) &= 0 \quad t < t_0 \\
u(t) &= 1 \quad t > t_0
\end{align*}
\]
Determining the type of solution

\[ \omega_0 = \sqrt{\frac{1}{L-C}} = \sqrt{\frac{1}{0.1\text{mH} \cdot 7\mu\text{F}}} = 37.8 \text{krad/s} \]

\[ \alpha = \frac{R}{2L} = \frac{3\text{k} \text{ohm}}{2 \cdot 0.1\text{mH}} = 1.5 \times 10^7 \frac{1}{\text{s}} \]

\[ \alpha > \omega_0 \text{ overdamped} \]

\[ f_0 = \frac{1}{2\pi} \omega_0 \]

\[ s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -47.9 \frac{1}{\text{s}} \]

\[ s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -3 \times 10^7 \frac{1}{\text{s}} \]

Note that this calculation can be done before we have even identified which voltage or current will be should be used when writing the solution to the 2\text{nd} order equation.
Boundary Conditions

In d.c. steady state, the capacitor will act like an open circuit ($\infty \Omega$) and the inductor will act as a short circuit (0 Ω).

The initial and final conditions for the voltages and currents are:

<table>
<thead>
<tr>
<th></th>
<th>Initial</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_R$</td>
<td>0 mA</td>
<td>0 mA</td>
</tr>
<tr>
<td>$V_R$</td>
<td>0 V</td>
<td>0 V</td>
</tr>
<tr>
<td>$i_C$</td>
<td>0 mA</td>
<td>0 mA</td>
</tr>
<tr>
<td>$V_C$</td>
<td>5 V</td>
<td>0 V</td>
</tr>
<tr>
<td>$i_L$</td>
<td>0 mA</td>
<td>0 mA</td>
</tr>
<tr>
<td>$V_L$</td>
<td>0 V</td>
<td>0 V</td>
</tr>
</tbody>
</table>
Because $V_c$ changed between initial and final conditions

$$V_c(t) = A_1 e^{-s_1 t} + A_2 e^{-s_2 t} + V_{c\text{final}}$$

$t > t_0 = 0\text{s}$

$$i_c(t) = \frac{C}{dt} V_c(t)$$

$$= \begin{bmatrix} -s_1 A_1 e^{-s_1 t} & -s_2 A_2 e^{-s_2 t} \end{bmatrix} C$$

Use initial conditions

$V_c(0\text{s}) = SV = A_1 e^{-s_1(0)} + A_2 e^{-s_2(0)} + OV$

$SV = A_1 + A_2$
You find that $A_1 = 5 \text{ V}$ (3 sig. figs.) and $A_2 = -7.97 \times 10^{-6} \text{ V} = -7.97 \mu\text{V}$

\[ v_C(t) = 5V e^{-47.8s^{-1}t} - 7.97\mu\text{Ve}^{-30Ms^{-1}t} \]