Examples:
Equivalent Impedance
Equivalent Impedance Calculation

• Suppose you were asked to find the current flowing out of the power supply. One way to calculate this is to find the equivalent impedance of the circuit and then apply Ohm’s Law.
\[
i = \frac{1 \text{V} \angle 0^\circ}{Z_{\text{eq}}}
\]
Frequency dependent solution

\[ 20\zeta + 10j\zeta \]

\[ -5\zeta \]

\[ 15\zeta \]

\[ 10\zeta \]

\[ Z_{eq} \]
\[
\left[ \frac{1}{20n+10j} + \frac{1}{-5j} \right]^{-1} = \frac{(20n+10j)(-5j)}{20n+10j-5j}
\]

\[
= \frac{1.18n - 5.29j}{2} = 1.18n - 20.29j
\]
\[
\left[ \frac{1}{1.18 \pi - 20.29 \, j} + \frac{1}{15 \, j} \right]^{-1}
\]

\[= 2.77 \pi - 7.85 \, j \, \pi \]

\[\Leftarrow Zeq = 12.8 \pi - 7.85 \, j \, \pi \]
Zeq

\[ i = \frac{1V}{15 \Omega} \angle -31.6^\circ = 66.7 \text{mA} \angle +31.6^\circ \]

\[ 12.8 \Omega - 7.85 \text{j} \Omega \]

\[ \frac{12.8 \Omega^2 + (-7.85)^2}{\sqrt{12.8 \Omega^2 + (-7.85)^2}} \angle \text{atan} \left[ \frac{-7.85}{12.8} \right] \]

\[ = 15.0 \Omega \angle -31.6^\circ \]
Power dissipated by circuit

\[ P = V_i = 1V \angle 0^\circ \cdot [66.7\text{mA} \angle +31.6^\circ] \]
\[ = 66.7\text{mW} \angle 31.6^\circ \]

This power must also be the power generated by the voltage source.

\[ P_{1V} = -66.7\text{mW} \angle 31.6^\circ = 66.7\text{mW} \angle -148.4^\circ \]
Compensation

• For power systems, it is best if the current flowing out of the power supply has the same phase angle as the voltage (i.e., the current and voltage from the power supply are in phase). In this case, that would mean that the phase angle of the current out of the power supply is 0°. This is known as power factor correction (PFC).
• For the next example, we would like to add an impedance in parallel with the voltage supply to force the equivalent impedance to have only a real component.
  – The additional impedance will cause the imaginary component to be zero.
  – Another way to say this is that the additional component causes \( Z_{eq} \) to have a phase angle of 0° so the phase angle of the current has the same phase angle as the voltage when we apply Ohm’s Law

• Note that an additional impedance can also be added in series with the voltage source, but it will have a different value.
Determine $Z_{\text{balance}}$
\[
\frac{1}{12.8\angle-7.85\text{j} \Omega} + \frac{1}{z_{\text{balance}}} = \text{Real component}
\]

\[
\left[\frac{1}{12.8\angle-7.85\text{j} \Omega} \right] = \frac{1}{0.0568\Omega + 0.0349\text{j} \Omega - 0.0349\text{j} \Omega}
\]

\[
z_{\text{balance}} = \frac{1}{0.0349\text{j} \Omega} = +28.7\text{j} \Omega
\]

only works at one frequency