Voltage and Current Division

Impedance and Admittance
Objective of Lecture

- Explain mathematically how a voltage that is applied to components in series and how a current that enters the a node shared by components in parallel are distributed among the components.
  - Chapter 9.7 Fundamentals of Electric Circuits
  - Chapters 15.4, 15.6, 16.3, 16.5, 17.2, 17.5, 17.7 Principles of Electric Circuits
  - Chapters 10.3, 10.5-6, 12.3, 12.5-6, 13.2, and 13.5 Electric Circuit Fundamentals
Voltage Dividers

Impedances in series share the same current
Voltage Dividers

From Kirchhoff’s Voltage Law and Ohm’s Law

\[ 0 = -V_s + V_1 + V_2 \]

\[ V_1 = IZ_1 \quad \text{and} \quad V_2 = IZ_2 \]

Therefore,

\[ V_2 = \frac{V_1}{Z_1} Z_2 \]

\[ V_1 = \frac{Z_1}{Z_1 + Z_2} V_s \]

\[ V_2 = \frac{Z_2}{Z_1 + Z_2} V_s \]
Voltage Division

The voltage associated with one impedance $Z_n$ in a chain of multiple impedances in series is:

$$V_n = \left[ \frac{Z_n}{\sum_{s=1}^{S} Z_s} \right] V_{\text{total}} \quad \text{or} \quad V_n = \left[ \frac{Z_n}{Z_{\text{eq}}} \right] V_{\text{total}}$$

where $V_{\text{total}}$ is the total of the voltages applied across the impedances.
Voltage Division

- Because of changes in phase angle of the voltage that occur with inductors and capacitors, the calculation of the percentage of the total voltage associated with a particular impedance, $Z_n$, is not directly related to the percentage of the magnitude of that particular impedance, $Z_n$, relative to the total equivalent impedance, $Z_{eq}$.
  - $Z_n = Z_n \angle \phi_n$
  - $Z_{eq} = Z_{eq} \angle \phi_{eq}$
Current Division

All components in parallel share the same voltage
Current Division

From Kirchhoff’s Current Law and Ohm’s Law

\[ 0 = -I_S + I_1 + I_2 + I_3 \]

\[ V_S = I_1 Z_1 \]

\[ V_S = I_2 Z_2 \]

\[ V_S = I_3 Z_3 \]
Current Division

\[ I_1 = \frac{Z_2 \parallel Z_3}{Z_1 + Z_2 \parallel Z_3} \cdot I_s \]

\[ I_2 = \frac{Z_1 \parallel Z_3}{Z_2 + Z_1 \parallel Z_3} \cdot I_s \]

\[ I_3 = \frac{Z_1 \parallel Z_2}{Z_3 + Z_1 \parallel Z_2} \cdot I_s \]
Current Division

where \( Z_{eq} = Z_2 \parallel Z_3 = \frac{Z_2Z_3}{Z_2 + Z_3} \) and \( I_1 = \frac{Z_{eq}}{Z_1 + Z_{eq}} I_s \)
Current Division

The current associated with one component $Z_i$ in parallel with one other component is:

$$I_1 = \left[ \frac{Z_2}{Z_1 + Z_2} \right] I_{\text{total}}$$

The current associated with one component $Z_m$ in parallel with two or more components is:

$$I_m = \left[ \frac{Z_{\text{eq}}}{Z_m} \right] I_{\text{total}}$$

where $I_{\text{total}}$ is the total of the currents entering the node shared by the components in parallel.
Summary

- The equations used to calculate the voltage across a specific component $Z_n$ in a set of components in series are:

$$V_n = \left[ \frac{Z_n}{Z_{eq}} \right] V_{total}$$

$$V_n = \left[ \frac{Y_{eq}}{Y_n} \right] V_{total}$$

- The equations used to calculate the current flowing through a specific component $Z_m$ in a set of components in parallel are:

$$I_m = \frac{Z_{eq}}{Z_m} I_{total}$$

$$I_m = \frac{Y_m}{Y_{eq}} I_{total}$$