Nonsinusoidal Waveforms
Objective of Lecture

- Introduce several nonsinusoidal waveforms including
  - Impulse function
  - Step function
  - Ramp function
  - Convolutions
    - Pulse and square waveforms
    - Sawtooth and triangular waveforms
- Discuss how to obtain a mathematical equation to describe a convoluted waveform.
- Explain what a composite wavefunction is and define fundamental and harmonic frequencies.
Nonsinusoidal Waveforms

- Not all signals in electrical and computer engineering are sinusoidal.
  - Most digital systems use square waveforms. Although at high switching speeds, these waveforms are starting to look trapezoidal.
  - Most bioelectric signals are nonsinusoidal, many are composite ramp functions.
  - When a switch is opened or closed, the time required for the signals to return to steady state is accompanied by sinusoidal and nonsinusoidal transients.
Singularity Functions

- Are discontinuous or have discontinuous derivatives.
  - Also known as switching functions. They are:
    - Impulse function
    - Unit function
    - Ramp functions
- Combinations of these functions can be used to describe complex waveforms.
  - Pulse function
  - Square waveform
  - Triangular waveform
  - Sawtooth waveform
Unit Step Function

\[ u(t - t_o) = \begin{cases} 
0 & t < t_o \\
1 & t > t_o
\end{cases} \]

\[ V(t) = V_o u(t - t_o) \]
Unit Impulse Function

- An impulse function is the derivative of a unit step function.

\[ \delta(t - t_o) = \frac{du(t - t_o)}{dt} \]

\[ \delta(t - t_o) = \begin{cases} 
0 & t < t_o \\
\infty & t = t_o \\
0 & t > t_o 
\end{cases} \]

\[ \int_{-\infty}^{+\infty} \delta(t - t_o)dt = \int_{t_o^-}^{t_o^+} \delta(t - t_o)dt = 1 \]
Unit Ramp Function

A unit ramp function is the result of the integration of a unit step function.

\[ r(t - t_o) = \int_{-\infty}^{t} u(t - t_o) \, dt = (t - t_o)u(t - t_o) \]

\[ r(t - t_o) = \begin{cases} 
0 & t < t_o \\
(t - t_o) & t > t_o 
\end{cases} \]
Unit Ramp Function

\[ r(t-t_0) \]

Slope = 1
Pulse Waveform
Convolution of Two Unit Step Functions

\[ u(t - t_o) \cdot u(t_1 - t) \]

or

\[ u(t - t_o)[1 - u(t - t_1)] \]
Square Waveform

$$3\left[u(t-t_0) - 2u(t-t_1) + u(t-t_2)\right]$$
Sawtooth Waveform

- This is a ramp function that is discontinued at time $t = t_1$. To discontinue a ramp function, the function must be multiplied by a second function that doesn’t disturb the ramp when $t < t_1$, but the result of the multiplication is zero for $t \geq t_1$. 
Sawtooth Waveform is formed by multiplying the Ramp Function times the Unit Step Function

\[ \frac{1}{t_1 - t_o} r(t - t_o) \cdot u(t_1 - t) \]
Triangular Waveform

$$\frac{4}{t_1 - t_0} \left[ r(t - t_0) - 2r(t - t_1) + r(t - t_2) \right] \quad \text{if } t_1 - t_0 = t_2 - t_1$$
Composite Waveforms

- Any nonsinusoidal function can be expressed as a sum (or composite) of multiple sinusoidal functions.
  - The sinusoids are related as the frequencies of the sinusoids are integral multiples of some base frequency, known as the fundamental frequency.
  - The higher frequencies are known as harmonics.
    - For even and odd harmonics, the multiplier is an even and odd integer, respectively.
    - The multiplier for $2^{nd}$ and $3^{rd}$ harmonics is 2 and 3, respectively.
Infinite Square Wave

- An infinite square wave is a weighted sum of a fundamental sinusoid and its odd harmonics.

\[ V(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \left\{ \frac{1}{2n-1} \sin \left[ \frac{(2n-1)\pi t}{T} \right] \right\} \]

- T is the period of the square waveform.
- The fundamental frequency, \( f_o = 1/T \).
Infinite Square Wave (con’t)

Fundamental
2nd harmonic
3rd harmonic

n = 20

T = 1 s, \( f_0 = 1 \text{Hz} \)
Summary

- Several unit functions were described and their mathematical functions were given.
  - The convolution of the unit functions can be used to form more complex functions.
- Composite waveforms are summations of sinusoidal waves, which is an alternative method to describe complex functions.