RC and RL Circuits

1st Order Circuits
Objective of the Lecture

- Explain the transient response of a RC circuit
  - As the capacitor stores energy when there is:
    - a transition in a unit step function source, $u(t-t_o)$
    - or a voltage or current source is switched into the circuit.

- Explain the transient response of a RL circuit
  - As the inductor stores energy when there is:
    - a transition in a unit step function source, $u(t-t_o)$
    - or a voltage or current source is switched into the circuit.

Also known as a forced response to an independent source
RC Circuit

$I_C = 0A$ when $t < t_0$

$V_C = 0V$ when $t < t_0$

Because $I_1 = 0A$ (replace it with an open circuit).
RC Circuit

- Find the final condition of the voltage across the capacitor.
  - Replace C with an open circuit and determine the voltage across the terminal.

\[ I_C = 0A \text{ when } t \sim \infty \text{ s} \]
\[ V_C = V_R = I_1R \text{ when } t \sim \infty \text{ s} \]
RC Circuit

In the time between $t_0$ and $t = \infty$ s, the capacitor stores energy and currents flow through R and C.

\[
V_C = V_R
\]

\[
I_C = C \frac{dV_C}{dt}
\]

\[
I_R = \frac{V_R}{R}
\]

\[
I_R + I_C - I_1 = 0
\]

\[
\frac{V_C}{R} + C \frac{dV_C}{dt} - I_1 = 0
\]

\[
V_C(t) = RI_1 \left[1 - e^{-\frac{t-t_0}{\tau}}\right] \quad \tau = RC
\]
RL Circuit (con’t)

- Initial condition is not important as the magnitude of the voltage source in the circuit is equal to 0V when \( t \leq t_0 \).
  - Since the voltage source has only been turned on at \( t = t_0 \), the circuit at \( t \leq t_0 \) is as shown below.
  - As the inductor has not stored any energy because no power source has been connected to the circuit as of yet, all voltages and currents are equal to zero.
RL Circuit

- So, the final condition of the inductor current needs to be calculated after the voltage source has switched on.
- Replace L with a short circuit and calculate $I_L(\infty)$. 
Final Condition

\[ V_L(\infty) = 0V \]
\[ I_L(\infty) = I_R \]
\[ I_R = \frac{V_1}{R} \]
RL Circuit

\[ \frac{dI_L}{dt} + RI_R - V_1 = 0 \]

\[ \frac{dI_L}{dt} + \frac{R}{L} I_L - \frac{V_1}{L} = 0 \]

\[ I_L(t) = \frac{V_1}{R} \left[ 1 - e^{-(t-t_0)/\tau} \right] \]

\[ \tau = \frac{L}{R} \]

\[ -V_1 + V_L + V_R = 0 \]

\[ I_L = I_R = \frac{V_R}{R} \]

\[ V_L = L \frac{dI_L}{dt} \]
Electronic Response

- Typically, we say that the currents and voltages in a circuit have reached steady-state once \( 5\tau \) have passed after a change has been made to the value of a current or voltage source in the circuit.
  - In a circuit with a forced response, percentage-wise how close is the value of the voltage across a capacitor in an RC circuit to its final value at \( 5\tau \)?
Complete Response

- Is equal to the natural response of the circuit plus the forced response
  - Use superposition to determine the final equations for voltage across components and the currents flowing through them.
Example #1

- Suppose there were two unit step function sources in the circuit.
Example #1 (con’t)

- The solution for $V_c$ would be the result of superposition where:
  - $I_2 = 0A$, $I_1$ is left on
    - The solution is a forced response since $I_1$ turns on at $t = t_1$
  - $I_1 = 0A$, $I_2$ is left on
    - The solution is a natural response since $I_2$ turns off at $t = t_2$
Example #1 (con’t)

\[ V_C(t) = 0V \quad \text{when } t < t_1 \]

\[ V_C(t) = RI_1 \left[ 1 - e^{-\frac{(t-t_1)}{RC}} \right] \quad \text{when } t > t_1 \]
Example #1 (con’t)

\[ V_C(t) = -RI_2 \quad \text{when } t < t_2 \]

\[ V_C(t) = -RI_2 e^{\frac{(t-t_2)}{RC}} \quad \text{when } t > t_2 \]
Example #1 (con’t)

- If $t_1 < t_2$

$$V_C(t) = 0V - RI_2$$

when $t < t_1$

$$V_C(t) = RI_1 \left[ 1 - e^{-\frac{(t-t_1)}{RC}} \right] - RI_2$$

when $t_1 < t < t_2$

$$V_C(t) = RI_1 \left[ 1 - e^{-\frac{(t-t_1)}{RC}} \right] - RI_2 e^{-\frac{(t-t_2)}{RC}}$$

when $t > t_2$
General Equations

When a voltage or current source changes its magnitude at \( t = 0 \) s in a simple RC or RL circuit.

Equations for a simple RC circuit

\[
V_C(t) = V_C(\infty) + \left[ V_C(0) - V_C(\infty) \right] e^{-t/\tau}
\]

\[
I_C(t) = \frac{C}{\tau} \left[ V_C(\infty) - V_C(0) \right] e^{-t/\tau}
\]

\( \tau = RC \)

Equations for a simple RL circuit

\[
I_L(t) = I_L(\infty) + \left[ I_L(0) - I_L(\infty) \right] e^{-t/\tau}
\]

\[
V_L(t) = \frac{L}{\tau} \left[ I_L(\infty) - I_L(0) \right] e^{-t/\tau}
\]

\( \tau = L / R \)
MATLAB

Needed to Complete HW # 21
How to renew your MatLAB license

http://swat.eng.vt.edu/Matlabtutorial.html

If you would like to contact them directly, SWAT is located at:
2080 Torgersen Hall
Office Hours: 12:00pm to 4:00pm Monday - Friday
Phone: (540) 231-7815
E-mail: swat@vt.edu
Introductory Tutorials

- MathWorks (www.mathworks.com) has
  - On-line tutorials including A Very Elementary MATLAB Tutorial
  - Videos (look at the ones below Getting Started)
  - Worked examples (further down the demos page)
- Textbook has a MatLAB tutorial in Appendix E.
Summary

- The final condition for:
  - the capacitor voltage \( V_o \) is determined by replacing the capacitor with an open circuit and then calculating the voltage across the terminals.
  - The inductor current \( I_o \) is determined by replacing the inductor with a short circuit and then calculating the current flowing through the short.

- The time constant for:
  - an RC circuit is \( \tau = RC \) and an RL circuit is \( \tau = L/R \)

- The general equations for the forced response of:
  - the voltage across a capacitor is \( V_C(t) = V_o \left[ 1 - e^{-(t-t_o)/\tau} \right] \) when \( t > t_o \)
  - the current through an inductor is \( I_L(t) = I_o \left[ 1 - e^{-(t-t_o)/\tau} \right] \) when \( t > t_o \)
Summary

- General equations when the magnitude of a voltage or current source in the circuit changes at $t = 0s$ for the:
  - voltage across a capacitor is $V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)]e^{-t/\tau}$
  - current through an inductor is $I_L(t) = I_L(\infty) + [I_L(0) - I_L(\infty)]e^{-t/\tau}$

- Superposition should be used if there are multiple voltage and/or current sources that change the magnitude of their output as a function of time.