Source Transformation

Basis for Thevenin and Norton Equivalent Circuits
Objective of Lecture

- Describe the differences between ideal and real voltage and current sources
- Demonstrate how a real voltage source and real current source are equivalent so one source can be replaced by the other in a circuit.
  - Chapter 4.4 Fundamentals of Electric Circuits
Voltage Sources

Ideal

- An ideal voltage source has no internal resistance.
  - It can produce as much current as is needed to provide power to the rest of the circuit.

Real

- A real voltage sources is modeled as an ideal voltage source in series with a resistor.
  - There are limits to the current and output voltage from the source.
Limitations of Real Voltage Source

\[
V_L = \frac{R_L}{R_L + R_S} V_S
\]

\[
I_L = V_L / R_L
\]
Voltage Source Limitations (con’t)

\[ R_L = 0\Omega \]

\[ V_L = 0V \]
\[ I_{L_{\text{max}}} = \frac{V_S}{R_S} \]
\[ P_L = 0W \]

\[ R_L = \infty\Omega \]

\[ V_L = V_S \]
\[ I_{L_{\text{min}}} = 0A \]
\[ P_L = 0W \]
Current Sources

Ideal

- An ideal current source has no internal resistance.
  - It can produce as much voltage as is needed to provide power to the rest of the circuit.

Real

- A real current source is modeled as an ideal current source in parallel with a resistor.
  - Limitations on the maximum voltage and current.
Limitations of Real Current Source

- Appear as the resistance of the load on the source approaches $R_s$. 

$$I_L = \frac{R_S}{R_L + R_S} I_S$$

$$V_L = I_L R_L$$
Current Source Limitations (con’t)

\( R_L = 0\Omega \)

\[
\begin{align*}
I_L &= I_S \\
V_{L_{\text{min}}} &= 0\text{V} \\
P_L &= 0\text{W}
\end{align*}
\]

\( R_L = \infty\Omega \)

\[
\begin{align*}
I_L &= 0\text{A} \\
V_{L_{\text{max}}} &= I_S R_S \\
P_L &= 0\text{W}
\end{align*}
\]
Electronic Response

- For a real voltage source, what is the voltage across the load resistor when $Rs = RL$?
- For a real current source, what is the current through the load resistor when $Rs = RL$?
Equivalence

- An equivalent circuit is one in which the i-v characteristics are identical to that of the original circuit.
  - The magnitude and sign of the voltage and current at a particular measurement point are the same in the two circuits.
Equivalent Circuits

- $R_L$ in both circuits must be identical.
  - $I_L$ and $V_L$ in the left circuit = $I_L$ and $V_L$ on the left

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1. **Real Voltage Source**

2. **Real Current Source**
Example #1

Find an equivalent current source to replace $V_s$ and $R_s$ in the circuit below.
Example #1 (con’t)

- Find $I_L$ and $V_L$.

\[
V_L = \frac{R_L}{R_L + R_S} V_S
\]

\[
V_L = \frac{6k\Omega}{6k\Omega + 3k\Omega} 18V = 12V
\]

\[
I_L = \frac{V_L}{R_L}
\]

\[
I_L = 12V / 6k\Omega = 2mA
\]

\[
P_{Vs} = P_L + P_{Rs}
\]

\[
P_{Vs} = 12V (2mA) + (18V - 12V)(2mA)
\]

\[
P_{Vs} = 36mW
\]
Example #1 (con’t)

- There are an infinite number of equivalent circuits that contain a current source.
  - If, in parallel with the current source, $R_s = \infty \Omega$
   - $R_s$ is an open circuit, which means that the current source is ideal.

\[
I_S = I_L \\
V_L = 2mA(6k\Omega) = 12V \\
P_L = V_L I_L = 12V (2mA) = 24mW \\
P_L = P_{I_S} = 24mW
\]
Example #1 (con’t)

- If $R_S = 20k\Omega$

\[
I_S = \frac{R_L + R_S}{R_S} I_L
\]

\[
I_S = \frac{6k\Omega + 20k\Omega}{20k\Omega} 2mA = 2.67mA
\]

\[
V_L = V_{Is} = I_L R_L = 12V
\]

\[
P_{Is} = P_L + P_{Rs} = V_L I_L + V_{Rs} I_{Rs}
\]

\[
P_{Is} = 12V (2mA) + 12V (2.67mA - 2mA)
\]

\[
P_{Is} = 32.0mW
\]
Example #1 (con’t)

- If $R_S = 6k\Omega$

$$I_S = \frac{R_L + R_S}{R_S} I_L$$

$$I_S = \frac{6k\Omega + 6k\Omega}{6k\Omega} 2mA = 4mA$$

$$V_L = V_{Is} = I_L R_L = 12V$$

$$P_{Is} = P_L + P_{Rs} = V_L I_L + V_{Rs} I_{Rs}$$

$$P_{Is} = 12V (2mA) + 12V (4mA - 2mA)$$

$$P_{Is} = 48mW$$
Example #1 (con’t)

- If $R_S = 3k\Omega$

\[ I_S = \frac{R_L + R_S}{R_S} I_L \]

\[ I_S = \frac{6k\Omega + 3k\Omega}{3k\Omega} \cdot 2mA = 6mA \]

\[ V_L = V_{Is} = I_L R_L = 12V \]

\[ P_{Is} = P_L + P_{Rs} = V_L I_L + V_{Rs} I_{Rs} \]

\[ P_{Is} = 12V (2mA) + 12V (6mA - 2mA) \]

\[ P_{Is} = 72mW \]
Example #1 (con’t)

- Current and power that the ideal current source needs to generate in order to supply the same current and voltage to a load increases as $R_S$ decreases. Note: $Rs$ cannot be equal to $0\Omega$.
  - The power dissipated by $R_L$ is 50% of the power generated by the ideal current source when $R_S = R_L$. 
Example #2

- Find an equivalent voltage source to replace $I_S$ and $R_S$ in the circuit below.
Example #2 (con’t)

- Find $I_L$ and $V_L$.

\[
I_L = \frac{50\Omega}{300\Omega + 50\Omega} I_S
\]

\[
I_L = 0.714mA
\]

\[
V_L = I_L R_L
\]

\[
V_L = 0.714mA(300\Omega) = 0.214V
\]

\[
P_{Vs} = P_L + P_{Rs}
\]

\[
P_{Vs} = 0.214V (0.714mA) + 0.214V (5mA - 0.714mA)
\]

\[
P_{Vs} = 1.07mW
\]
Example #2 (con’t)

- There are an infinite number of equivalent circuits that contain a voltage source.
  - If, in series with the voltage source, $Rs = 0\Omega$
    - $Rs$ is a short circuit, which means that the voltage source is ideal.

\[
V_S = V_L = 0.214V \\
I_L = \frac{V_L}{R_L} = \frac{0.214V}{300\Omega} = 0.714mA \\
P_L = V_L I_L = 0.214V \times 0.714mA = 0.153mW \\
P_L = P_{Vs} = 0.153mW
\]
Example #2 (con’t)

- If $R_S = 50\Omega$

\[
V_S = \frac{R_L + R_S}{R_L} V_L \\
V_S = \frac{300\Omega + 50\Omega}{300\Omega} \times 0.214V = 0.25V
\]

\[
I_L = I_{V_S} = V_L / R_L = 0.714mA
\]

\[
P_{V_S} = P_L + P_{Rs} = V_L I_L + V_{Rs} I_{Rs}
P_{V_S} = 0.214V \times 0.714A + (0.25V - 0.214V)(0.714mA)
P_{V_S} = 0.179mW
\]
Example #2 (con’t)

- If $R_S = 300\Omega$

\[
V_S = \frac{R_L + R_S}{R_L} V_L
\]

\[
V_S = \frac{300\Omega + 300\Omega}{300\Omega} 0.214V = 0.418V
\]

\[
I_L = I_{Vs} = V_L / R_L = 0.714mA
\]

\[
P_{Vs} = P_L + P_{Rs} = V_L I_L + V_{Rs} I_{Rs}
\]

\[
P_{Vs} = 0.214V(0.714A)
\]

\[
+ (0.418V - 0.214V)(0.714mA)
\]

\[
P_{Vs} = 0.306mW
\]
Example #2 (con’t)

- If \( R_S = 1k\Omega \)

\[
V_S = \frac{R_L + R_S}{R_L} V_L
\]

\[
V_S = \frac{300\Omega + 1k\Omega}{300\Omega} 0.214V = 0.927V
\]

\[
I_L = I_{V_S} = \frac{V_L}{R_L} = 0.714mA
\]

\[
P_{V_S} = P_L + P_{Rs} = V_L I_L + V_{Rs} I_{Rs}
\]

\[
P_{V_S} = 0.214V(0.714A) + (0.927V - 0.214V)(0.714mA)
\]

\[
P_{V_S} = 0.662mW
\]
Example #2 (con’t)

- Voltage and power that the ideal voltage source needs to supply to the circuit increases as $R_S$ increases. $R_s$ can not be equal to $\infty \Omega$.
  - The power dissipated by $R_L$ is 50% of the power generated by the ideal voltage source when $R_S = R_L$. 
Summary

- An equivalent circuit is a circuit where the voltage across and the current flowing through a load $R_L$ are identical.
  - As the shunt resistor in a real current source decreases in magnitude, the current produced by the ideal current source must increase.
  - As the series resistor in a real voltage source increases in magnitude, the voltage produced by the ideal voltage source must increase.
  - The power dissipated by $R_L$ is 50% of the power produced by the ideal source when $R_L = R_S$. 