Use Nodal Analysis to solve for all of the branch currents and node voltages in the circuits below. Show all work.

Note that the solutions for the node voltage depend on which node you chose as ground. However, magnitude of the voltage drops and the currents will be unaffected by this choice.

1. 

\[ @V_a: \quad -2m + I_1 = 0 \]

\[ @V_b: \quad -I_1 + I_3 + 2m = 0 \]

\[ -2m + \frac{V_a - V_b}{5k} = 0 \quad (1) \]

\[ \frac{V_b-V_a}{5k} + \frac{V_b-V_d}{6k} + 2m = 0 \quad (2) \]

By substituting the first equation into the second, we get:

\[ -2m + \frac{V_b - V_d}{6k} + 2m = 0 \quad \text{or simply: } \quad V_b = V_d \]

Thus, \( I_3 = 0 \) A.

\[ @V_c: \quad 2m - 2m + I_2 = 0 \]

\[ 2m - 2m + \frac{V_c}{3k} = 0 \quad \text{or } \quad V_c = 0 \text{ V} \quad (3) \]

\[ @V_d: \quad -I_{bat} - I_3 + I_5 + I_6 = 0 \]

\[ -I_{bat} + \frac{V_d - V_b}{6k} + \frac{V_d - V_e}{1k} + \frac{V_d}{2k} = 0 \]

Notice that we cannot directly express \( I_{bat} \) in terms of node voltages. We will return to this shortly.
@V_d: \( I_{bat} + I_4 - I_5 = 0 \)

\[
I_{bat} + \frac{V_e}{8k} + \frac{V_e - V_d}{1k} = 0
\]

By plugging this equation into the previous one (@V_d):

\[
\frac{V_e}{8k} + \frac{V_e - V_d}{1k} + \frac{V_d - V_b}{6k} + \frac{V_d - V_e}{1k} + \frac{V_d}{2k} = 0
\]

Noting that \( V_b = V_d \) (from the equation @V_b), the above equation simplifies to:

\[
\frac{V_e}{8k} + \frac{V_d}{2k} = 0 \quad (4)
\]

Since the two equations established at \( V_d \) and \( V_e \) collapsed into one as a result of \( I_{bat} \), we require an additional equation to solve for the 5 unknowns. We obtain this equation by noting that:

\[
V_d - V_e = 10V \quad (5)
\]

From (4) and (5), we find \( V_e = -8 \) and \( V_d = 2 \).

Thus far, we have:

\[
V_b = 2V, \quad V_c = 0V, \quad V_d = 2V, \text{ and } V_e = -8V
\]

Finally, we find \( V_a \) using the equation @V_a:

\[
V_a = 2m(5k) + V_b = 12V
\]

Now we can find all of the currents from:
Note that the voltage drop \( V_a - V_c = 12 \) V while the voltage drop \( V_c - V_b = -2 \) V. This means that the current source on the left is generating power (-24 mW). However, the current source on the right is dissipating power (+4 mW). Just because the circuit has three power supplies in it, it does not mean that all of the power supplies are generating power. Power supplies may dissipate power as well as generate it. For example, a car battery is a d.c. voltage supply that generates power when you use it to start your car’s engine. However, it dissipates power as the alternator charges the battery while you drive.
By selecting the node below the 5 V source as ground, we can easily determine \( V_d \) as -5 V or:

\[
0 - V_d = 5 \rightarrow V_d = -5 \text{ V}
\]

Furthermore, we can solve for \( V_a \) by noting that:

\[
V_d - V_a = 3 \rightarrow -5 - V_a = 3 \text{ or } V_a = -8 \text{ V}
\]

Now, we are left with only 2 unknowns (\( V_b \) and \( V_c \)), and therefore 2 equations.

@\( V_b \):

\[
-I_5 + I_6 - I_7 = 0
\]

\[
\frac{V_b - V_c}{2k} + \frac{V_b + 5}{4k} + \frac{V_b + 8}{1k} = 0 \text{ or } \frac{V_b}{1k} + \frac{V_b}{4k} + \frac{V_b - V_c}{2k} = -9.25 \text{ m}
\]

@\( V_c \):

\[
I_3 + I_4 + I_5 - 1\text{ m} = 0
\]

\[
\frac{V_c}{10k} + \frac{V_c + 5}{1k} + \frac{V_c - V_b}{2k} - 1\text{ m} = 0 \text{ or } \frac{V_c - V_b}{2k} + \frac{V_c}{1k} + \frac{V_c}{10k} = -4 \text{ m}
\]

Solving the two equations yields: \( V_b = -6.59 \text{ V} \) and \( V_c = -4.56 \text{ V} \)

So we have:

\[
V_d = -8 \text{ V} \quad, \quad V_b = -6.59 \text{ V} \quad \text{and} \quad V_c = -4.56 \text{ V}
\]

The currents are then:

\[
I_1 = \frac{5}{3k} = 1.67 \text{ mA}, \quad I_2 = I_3 - I_1 = -2.12 \text{ mA} \quad I_3 = \frac{V_c}{10k} = -0.456 \text{ mA}
\]

\[
I_4 = \frac{V_c + 5}{1k} = 0.44 \text{ mA}, \quad I_5 = \frac{V_c - V_b}{2k} = 1.02 \text{ mA} \quad I_6 = \frac{V_b + 5}{4k} = -0.397 \text{ mA},
\]
In summary:

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Va</td>
<td>-8</td>
</tr>
<tr>
<td>Vb</td>
<td>-6.59</td>
</tr>
<tr>
<td>Vc</td>
<td>-4.56</td>
</tr>
<tr>
<td>(Vd=-5 V)</td>
<td></td>
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</tbody>
</table>

\[ I₇ = \frac{V_a - V_b}{1k} = -1.41 \text{ mA} \]
First, we can write by inspection:

\[ V_a - V_b = 8 \text{ V} \quad (1) \]
\[ V_d - V_c = 20 \text{ V} \quad (2) \]

Now, we apply KCL at each node:

@\( V_d \): \( I_1 + I_2 - I_1 + I_7 + I_3 - I_5 = 0 \)
Since \( I_1 \) cancels out:

\[ I_2 + I_7 + I_3 - I_5 = 0 \]
\[ \frac{V_a}{10} + \frac{V_a - V_c}{15} + \frac{V_a}{5} + \frac{V_a - V_e}{3} = 0 \]

Which simplifies to:

\[ \left( \frac{1}{10} + \frac{1}{5} + \frac{1}{3} + \frac{1}{15} \right) V_a + \frac{-V_c}{15} + \frac{-V_e}{3} = 0 \quad (3) \]

Next:

@\( V_b \): \(-I_1 + I_1 = 0\) or \(-I_1 + \frac{V_b - V_a}{2} = 0 \rightarrow I_1 = \frac{V_b - V_a}{2} \)

\[ I_1 = \frac{V_b - V_a}{2} \quad (4) \]
We can substitute in $I_7 = \frac{V_a - V_c}{15}$ from the previous equation and rearrange to get:

$$-\frac{1}{15}V_a + \frac{1}{15}V_c + \left(\frac{1}{2} + \frac{1}{3}\right)V_d - \frac{1}{2}V_e = -1 \quad (5)$$

Finally, 

$@V_e$: \[ I_5 - I_6 - 1 = 0 \]

$$\frac{V_e - V_a}{3} + \frac{V_e - V_d}{2} - 1 = 0$$

Which can be expressed as:

$$-\frac{1}{3}V_a - \frac{1}{2}V_d + \left(\frac{1}{2} + \frac{1}{3}\right)V_e = 1$$

From nodal analysis, we have found 5 equations and 5 unknowns that are summarized below:

1. \[ V_a - V_b = 8 \text{ V} \]
2. \[ -V_c + V_d = 20 \text{ V} \]
3. \[ \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{3} + \frac{1}{15}\right)V_a + \left(-\frac{V_c}{15}\right) + \frac{-V_e}{3} = 0 \]
4. \[ I_1 = \frac{V_b - V_a}{2} \text{ (not useful)} \]
5. \[ -\frac{1}{15}V_a + \frac{1}{15}V_c + \left(\frac{1}{2} + \frac{1}{3}\right)V_d - \frac{1}{2}V_e = -1 \]
6. \[ -\frac{1}{3}V_a - \frac{1}{2}V_d + \left(\frac{1}{2} + \frac{1}{3}\right)V_e = 1 \]

Notice that the nodal equations yielded 6 equations, but (4) is not useful as it contains $I_1$. As a result, we are left with 5 independent equations, which is adequate to find the 5 nodal voltages.
Solving the set of equations yields:

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_A )</td>
<td>-1.16 V</td>
</tr>
<tr>
<td>( V_B )</td>
<td>-9.16 V</td>
</tr>
<tr>
<td>( V_C )</td>
<td>-18.96 V</td>
</tr>
<tr>
<td>( V_D )</td>
<td>1.04 V</td>
</tr>
<tr>
<td>( V_E )</td>
<td>1.36 V</td>
</tr>
<tr>
<td>( V_f )</td>
<td>1.04 V</td>
</tr>
<tr>
<td>( I_7 )</td>
<td>1.19 A</td>
</tr>
</tbody>
</table>

\[ V_a = -1.16 \text{ V} \]
\[ V_b = -9.16 \text{ V} \]
\[ V_c = -18.96 \text{ V} \]
\[ V_d = 1.04 \text{ V} \]

\[ I_2 = -0.116 \text{ A} \]
\[ I_3 = -0.231 \text{ A} \]
\[ I_4 = 0.347 \text{ A} \]

\[ I_5 = 0.84 \text{ A} \]
\[ I_6 = 2 \text{ A} \]
\[ I_7 = 1.19 \text{ A} \]