Nodal Analysis

Examples
Example 1
First, find all of the nodes in the circuit.
There are four nodes in this circuit.
Next, label the nodes.
Pick a node to be your reference node.

- I will pick the node with the most connections.
- So, V0 will be the reference and I will set its voltage to be equal to 0 V.
Label the currents

- These are the currents that flow through all of the components including the voltage and current sources.
Use Kirchhoff’s Current Law

- Current entering a node must be equal to the current leaving a node.

- But you only need to use it on nodes V1, V2, and V3.
Node V1

\[ I_A = I_B + I_V \]
\[ I_V + I_C = 0A \]
Node V3

\[ I_D + I_I = I_C \]
Apply Ohm’s Law

- To rewrite currents in terms of node voltages.
  - The difference in the node voltages across a resistor is equal to its resistance times the current flowing through the resistor.
  - Ohm’s Law does not apply to current or voltage sources. So we cannot rewrite $Iv$ or $I\bar{I}$. 
\[ I_A = \frac{(0V - V1)}{4k\Omega} = -\frac{V1}{4k\Omega} \]
\[ I_B = \frac{(V1 - 0V)}{2k\Omega} = \frac{V1}{2k\Omega} \]
\[ I_C = \frac{(V_3 - V_2)}{3k\Omega} \]

\[ I_D = \frac{(0V - V_3)}{2k\Omega} = -\frac{V_3}{2k\Omega} \]
Substitute into the KCL equations

KCL Equations

Node V1: \( I_A = I_B + I_V \)

Node V2: \( I_V + I_C = 0 \text{A} \)

Node V3: \( I_C = I_D + I_I \)

KCL With Ohm’s Law

Node V1: \( \frac{-V_1}{4k\Omega} = \frac{V_1}{2k\Omega} + I_V \)

Node V2: \( I_V + \frac{V_3 - V_2}{3k\Omega} = 0 \text{A} \)

Node V3: \( \frac{V_3 - V_2}{3k\Omega} = \frac{V_3}{2k\Omega} + I_I \)
Gather other information from circuit

The difference in voltage between $V_2$ and $V_1$ is $5V$

$$V_2 - V_1 = 5V \quad \text{or} \quad V_2 = 5V + V_1$$
3 equations and three unknowns

\[ \text{Node V1: } \frac{-V_1}{4k\Omega} = \frac{V_1}{2k\Omega} + I_v \]

\[ \text{Node V2: } I_v + \frac{V_3 - (5V + V_1)}{3k\Omega} = 0A \]

\[ \text{Node V3: } \frac{V_3 - (5V + V_1)}{3k\Omega} = \frac{V_3}{2k\Omega} + 1mA \]
Solution methods

- Use MATLAB or other mathematical software package.
- Use a graphing calculator.
- Calculate this by hand.

- You will not have access to MATLAB or other mathematical software or a graphing calculator at the final exam. Make sure that you know how to find the solutions to a set of linear equations by hand.
Example 2: Wheatstone Bridge

- Used in to accurately and precisely determine value of one unknown component.
  - Many times the component’s value changes with an environmental parameter, which allows you to monitor the environmental parameter.
  - For example, a thermister is a resistor whose value is a known function of temperature. If we determine the value of the thermister, we can then calculate the temperature of the thermistor.
    - This technique is used to monitor temperature of furnaces, ovens, refrigerators, and other systems.
$V_{meas}$ is a function of $R_L$
Label nodes and let $VD = 0V$

- Note that $V_{meas}$ is the difference in voltage between nodes $V_C$ and $V_B$: $V_{meas} = V_C - V_B$
Label currents
Write Equations

KCL:

\[I_V = I_1 + I_2\]
\[I_1 = I_3\]
\[I_2 = I_L\]

Ohm's Law:

\[I_1 = \left( V_A - V_B \right) / R_1\]
\[I_2 = \left( V_A - V_C \right) / R_2\]
\[I_3 = \left( V_B - V_D \right) / R_3 = V_B / R_3\]
\[I_L = \left( V_C - V_D \right) / R_L = V_C / R_L\]

Extra Equations:

\[V_{in} = V_A - V_D = V_A\]
\[V_{meas} = V_C - V_B\]
Resistor values

- Usually in Wheatstone Bridge circuits,
  - $R_1 = R_2 = R_3 = R$

- Thus, the solution for $V_{\text{meas}}$ is:

$$V_{\text{meas}} = V_{\text{in}} \left[ \left( \frac{R_L}{R_L + R} \right) - \frac{1}{2} \right]$$

- When $R_L = R$, $V_{\text{meas}} = 0V$.

- When $R_L < R$, $V_{\text{meas}} < 0V$.

- When $R_L > R$, $V_{\text{meas}} > 0V$. 
Summary

- Nodal Analysis is used to calculate the voltage at every node in a circuit and the currents flowing out of or into the voltage sources using a set of simultaneous linear equations.
  - The currents flowing through each resistor can then be calculated using the equations written when applying Ohm’s Law.