RC and RL Filters
Which is a low pass RC filter?

Circuit A

Circuit B
Which is a low pass RC filter?

**Circuit B**

**Why?**

- \( V_o = V_{in} \) when \( Z_c \) is \(-j\infty \Omega\)
  - This happens when the capacitor acts like an open circuit, which is at 0 Hz (i.e., dc conditions)

- \( V_o = 0 \text{ V} \) when \( Z_c \) is \(-j0 \Omega\)
  - This happens when the capacitor acts like a short circuit, which is at \( \infty \text{ Hz} \).

- The input signal is passed on to the output at low frequencies.
Which is a low pass RL filter

Circuit A

Circuit B
Which is a low pass RL filter

Circuit B

Why?

- $V_o = V_{\text{in}}$ when $Z_L$ is $j0 \ \Omega$
  - This happens when the inductor acts like a short circuit, which is at 0 Hz (i.e., dc conditions)

- $V_o = 0 \ \text{V}$ when $Z_c$ is $j\infty \ \Omega$
  - This happens when the inductor acts like an open circuit, which is at $\infty$ Hz.

- Again, the input signal is passed to the output at low frequencies.
Voltage Transfer Characteristic

- Plot of $V_o$ as a function of $V_{in}$
  - The general shape of the voltage transfer characteristic for a low pass RC filter looks identical to that of a low pass RL filter.
  - The general shape of the voltage transfer characteristic for a high pass RC filter looks identical to that of a high pass RL filter.
Voltage Transfer Function

**Low Pass RC Filter**

\[
\frac{V_o}{V_{in}} = \frac{Z_C}{Z_R + Z_C} = \frac{-j}{\omega C} = \frac{1}{R - j\omega RC} = 1 + j\omega RC
\]

\[|H(\omega)| = \left| \frac{V_o}{V_{in}} \right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}\]

\[|H(\omega)|_{dB} = 20\log\left| \frac{V_o}{V_{in}} \right| = 20\log\left( \frac{1}{\sqrt{1 + (2\pi f RC)^2}} \right)\]

When \( |H(\omega)|_{dB} = -3\text{dB} \), \( Z_C = Z_R \) and \( f_c = \frac{1}{2\pi RC} \)

\[\theta_{V_o} - \theta_{Vin} = a \tan(-\omega RC) = -a \tan(\omega RC)\]

**Low Pass RL Filter**

\[
\frac{V_o}{V_{in}} = \frac{Z_R}{Z_R + Z_L} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R}
\]

\[|H(\omega)| = \left| \frac{V_o}{V_{in}} \right| = \frac{1}{\sqrt{1 + (\omega L/R)^2}}\]

\[|H(\omega)|_{dB} = 20\log\left| \frac{V_o}{V_{in}} \right| = 20\log\left( \frac{1}{\sqrt{1 + (2\pi f L/R)^2}} \right)\]

When \( |H(\omega)|_{dB} = -3\text{dB} \), \( Z_L = Z_R \) and \( f_c = \frac{1}{2\pi L/R} \)

\[\theta_{V_o} - \theta_{Vin} = a \tan(-\omega L/R) = -a \tan(\omega L/R)\]
Voltage Transfer Function

**High Pass RC Filter**

\[
\frac{V_o}{V_{in}} = \frac{Z_R}{Z_R + Z_C} = \frac{R}{R - \frac{j}{\omega C}} = \frac{j\omega RC}{1 + j\omega RC}
\]

\[|H(\omega)| = \left| \frac{V_o}{V_{in}} \right| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}\]

\[|H(\omega)|_{dB} = 20\log \left| \frac{V_o}{V_{in}} \right| = 20\log \left( \frac{2\pi fRC}{\sqrt{1 + (2\pi fRC)^2}} \right)\]

When \(|H(\omega)|_{dB} = -3\, dB\), \(Z_C = Z_R\) and \(f_c = \frac{1}{2\pi RC}\)

\[\theta_{V_o} - \theta_{V_{in}} = 90^\circ - \alpha \tan(\omega RC)\]

**High Pass RL Filter**

\[
\frac{V_o}{V_{in}} = \frac{Z_L}{Z_L + Z_R} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega L/R}{1 + j\omega L/R}
\]

\[|H(\omega)| = \left| \frac{V_o}{V_{in}} \right| = \frac{\omega L/R}{\sqrt{1 + (\omega L/R)^2}}\]

\[|H(\omega)|_{dB} = 20\log \left| \frac{V_o}{V_{in}} \right| = 20\log \left( \frac{2\pi fL/R}{\sqrt{1 + (2\pi fL/R)^2}} \right)\]

When \(|H(\omega)|_{dB} = -3\, dB\), \(Z_L = Z_R\) and \(f_c = \frac{1}{2\pi L/R}\)

\[\theta_{V_o} - \theta_{V_{in}} = 90^\circ - \alpha \tan(\omega L/R)\]
Decibels (dB)

\[
dB = 10\log\left( \frac{P_{out}}{P_{in}} \right)
\]

\[
dB = 10\log\left( \frac{V_o^2}{V_in^2} \right) = 10\log\left( \frac{V_o^2}{V_in^2} \right) = 20\log\left( \frac{V_o}{V_in} \right)
\]

\[
dBm = 10\log\left( \frac{P_o}{P_{in}} \right) \text{ where } P_{in} = 1mW
\]
Voltage Transfer Characteristics

Low Pass Filter

High Pass Filter
Plot in dB for Low Pass Filter

Frequency (Hz) vs. dB

-3dB

-20 dB/decade rolloff

$f_c$
Phase Angle

Low Pass Filter

High Pass Filter