Voltage and Current Division

Impedance and Admittance
Objective of Lecture

- Explain mathematically how a voltage that is applied to components in series and how a current that enters the a node shared by components in parallel are distributed among the components.

Chapter 8.7 in Basic Engineering Circuit Analysis by Irwin and Nelms
Voltage Dividers

Impedances in series share the same current
Voltage Dividers

From Kirchhoff’s Voltage Law and Ohm’s Law

\[ \begin{align*}
0 &= -V_s + V_1 + V_2 \\
V_1 &= IZ_1 \quad \text{and} \quad V_2 = IZ_2 \\
\text{Therefore,} \quad V_2 &= \frac{V_1}{Z_1}Z_2
\end{align*} \]

\[ \begin{align*}
V_1 &= \frac{Z_1}{Z_1 + Z_2} V_s \\
V_2 &= \frac{Z_2}{Z_1 + Z_2} V_s
\end{align*} \]
Voltage Division

The voltage associated with one impedance $Z_n$ in a chain of multiple impedances in series is:

$$V_n = \left[ \frac{Z_n}{\sum_{s=1}^{S} Z_s} \right] V_{\text{total}} \quad \text{or} \quad V_n = \left[ \frac{Z_n}{Z_{eq}} \right] V_{\text{total}}$$

where $V_{\text{total}}$ is the total of the voltages applied across the impedances.
Voltage Division

- Because of changes in phase angle of the voltage that occur with inductors and capacitors, the calculation of the percentage of the total voltage associated with a particular impedance, \( Z_n \), is not directly related to the percentage of the magnitude of that particular impedance, \( Z_n \), relative to the total equivalent impedance, \( Z_{eq} \).
- \( Z_n = Z_n \angle \phi_n \)
- \( Z_{eq} = Z_{eq} \angle \phi_{eq} \)
Current Division

All components in parallel share the same voltage
Current Division

From Kirchhoff’s Current Law and Ohm’s Law

\[ 0 = -I_s + I_1 + I_2 + I_3 \]

\[ V_s = I_1 Z_1 \]
\[ V_s = I_2 Z_2 \]
\[ V_s = I_3 Z_3 \]
Current Division

\[ I_1 = \frac{Z_2 \parallel Z_3}{Z_1 + Z_2 \parallel Z_3} I_s \]

\[ I_2 = \frac{Z_1 \parallel Z_3}{Z_2 + Z_1 \parallel Z_3} I_s \]

\[ I_3 = \frac{Z_1 \parallel Z_2}{Z_3 + Z_1 \parallel Z_2} I_s \]
Current Division

\[ S \neq 1 \neq \frac{1}{3^2} \]

where \( Z_{eq} = Z_2 \parallel Z_3 = \frac{Z_2 Z_3}{Z_2 + Z_3} \) and \( I_1 = \frac{Z_{eq}}{Z_1 + Z_{eq}} I_S \)
Current Division

The current associated with one component $Z_l$ in parallel with one other component is:

$$ I_1 = \left[ \frac{Z_2}{Z_1 + Z_2} \right] I_{\text{total}} $$

The current associated with one component $Z_m$ in parallel with two or more components is:

$$ I_m = \left[ \frac{Z_{\text{eq}}}{Z_m} \right] I_{\text{total}} $$

where $I_{\text{total}}$ is the total of the currents entering the node shared by the components in parallel.
Summary

- The equations used to calculate the voltage across a specific component $Z_n$ in a set of components in series are:

  \[ V_n = \left[ \frac{Z_n}{Z_{eq}} \right] V_{total} \]

  \[ V_n = \left[ \frac{Y_{eq}}{Y_n} \right] V_{total} \]

- The equations used to calculate the current flowing through a specific component $Z_m$ in a set of components in parallel are:

  \[ I_m = \frac{Z_{eq}}{Z_m} I_{total} \]

  \[ I_m = \frac{Y_m}{Y_{eq}} I_{total} \]