Audio circuits, such as the circuits used to pick up sounds with the microphone or to emit sounds from the speakers, cover a frequency range from 10 Hz to 50 kHz for high fidelity systems or a range of considerably less for voice-only applications such as cell phones. Systems where high speed and large amounts of data are transmitted, such as in the arithmetic logic unit (ALU) on your computer motherboard, fiber optic communications used in cell and land-line phone systems, and the Ethernet connections from your computer to the internet, are designed to operate a much higher frequencies – up to 100 GHz. All of these circuits use capacitors, inductors, and resistors so the impedance of each component at these different frequencies must be determined.

1. Calculate the impedance of ideal capacitors at the frequencies listed below. Write each answer in rectangular coordinates and then convert to phaser notation.

   \[ Z_c = -\frac{j}{\omega C} \] \[ \Omega = \frac{1}{\omega C} \Omega \angle -90^\circ \]

   a. 2.2 nF at 12 kHz
      \[ Z_c = -j \times 6.03 \text{ k}\Omega = 6.03 \text{ k}\Omega \angle -90^\circ \]

   b. 2.2 nF at 40 GHz
      \[ Z_c = -j 1.81 \text{ m}\Omega = 1.81 \text{ m}\Omega \angle -90^\circ \]

   c. 47 \( \mu \)F at 12 kHz
      \[ Z_c = -j 0.282 \Omega = 0.282 \Omega \angle -90^\circ \]

   d. 47 \( \mu \)F at 40 GHz
      \[ Z_c = -j 84.7 \text{ n}\Omega = 84.7 \text{ n}\Omega \angle -90^\circ \]

2. Calculate the impedance of ideal inductors at the frequencies listed below. Write each answer in rectangular coordinates and then convert to phaser notation.

   \[ Z_L = j\omega L \Omega \angle 90^\circ \]

   a. 30 nH at 12 kHz
      \[ Z_L = j2.26 \text{ m}\Omega = 2.26 \text{ m}\Omega \angle 90^\circ \]

   b. 30 nH at 40 GHz
      \[ Z_L = j7.54 \text{ k}\Omega = 7.54 \text{ k}\Omega \angle 90^\circ \]

   c. 1 mH at 12 kHz
      \[ Z_L = j75.4 \Omega = 75.4 \Omega \angle 90^\circ \]

   d. 1 mH at 40 GHz
      \[ Z_L = j0.251 \text{ G}\Omega = 0.251 \text{ G}\Omega \angle 90^\circ \]

3. The model for a real capacitor takes into account the current that leaks between the two electrodes (because all materials, even insulators, will conduct current) and the inductance of the wires that are used to connect the capacitor to the rest of the circuit.

   a. Determine an equation for the equivalent impedance of the capacitor model shown to right using rectangular coordinates.

   Denoting the impedance of the inductor and capacitor as \( Z_L \) and \( Z_c \), respectively:
   \[ R_{eq} = R|Z_c + Z_L| \]

   which becomes:
   \[ R_{eq} = \frac{RZ_c}{R + Z_c} + Z_L = \frac{R}{\frac{1}{f\omega C}} + j\omega L = \frac{R}{1 + j\omega CR} + j\omega L \]
To manipulate the expression into rectangular coordinates (real and imaginary parts), we can utilize the complex conjugate:

\[ R_{eq} = \frac{R}{1 + j\omega CR} \left( \frac{1 - j\omega CR}{1 - j\omega CR} \right) + j\omega L = \frac{R - j\omega CR^2}{1 + (\omega CR)^2} + j\omega L \]

Finally, the rectangular coordinates are:

\[ R_{eq} = \frac{R}{1 + (\omega CR)^2} + j \left( \frac{\omega CR^2}{1 + (\omega CR)^2} \right) \]

b. If \( C = 2.2 \text{ pF} \), \( R = 100 \ \text{M} \Omega \), and \( L = 30 \text{ nH} \), find the equivalent impedance of the practical capacitor at a frequency:
   i. \( f = 12 \text{ kHz} \)
   \[ R_{eq} = 0.362 \ \text{M} \Omega - j6.01 \ \text{M} \Omega \]

   ii. \( f = 40 \text{ GHz} \)
   \[ R_{eq} = 32.8 \ \text{n} \Omega + j7.54 \ \text{k} \Omega \]

4. The model for a real inductor takes into account the resistance of the wire wound around the core and the parasitic capacitance created between each of the coils of wire.
   a. Determine an equation for the equivalent impedance of the inductor model shown to the right using rectangular coordinates.

   \[ R_{eq} = (R + Z_L)||Z_C = \frac{(R + Z_L)Z_C}{R + Z_L + Z_C} = \frac{(R + j\omega L) \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} \]

   Simplifying the expression further:

   \[ R_{eq} = \frac{R + j\omega L}{j\omega C(R + j\omega L) + 1} = \frac{R + j\omega L}{(1 - \omega^2 LC) + j\omega RC} \]

   Again utilizing the complex conjugate to eliminate the complex denominator:

   \[ R_{eq} = \frac{R + j\omega L}{(1 - \omega^2 LC) + j\omega RC} \left( \frac{1 - \omega^2 LC - j\omega RC}{1 - \omega^2 LC - j\omega RC} \right) \]

   After a few simplifications, we arrive at:

   \[ R_{eq} = \frac{R}{C^2 L^2 \omega^4 + \omega^2 (C^2 R^2 - 2CL) + 1} + j \frac{(L - CR^2)\omega - CL^2 \omega^3}{C^2 L^2 \omega^4 + \omega^2 (C^2 R^2 - 2CL) + 1} \]

b. If \( L = 1 \text{ mH} \), \( R = 50 \ \Omega \), and \( C = 47 \ \text{m} \mu \text{F} \),
   i. at what frequency, \( f \), are the magnitudes of the real and imaginary components of the impedance equal?

   Equating the magnitudes of the real and imaginary:

   \[ R = \left| (L - CR^2)\omega - CL^2 \omega^3 \right| \]

   which yields \( f = 68.3 \text{ Hz} \)

   ii. at what frequency, \( f \), are the magnitudes of the imaginary component of the impedance equal zero? \( f = 0 \text{ Hz} \) and \( f = \infty \text{ Hz} \).