Capacitors
Energy Storage Devices
Objective of Lecture

- Describe the construction of a capacitor and how charge is stored.
- Introduce several types of capacitors
- Discuss the electrical properties of a capacitor
  - The relationship between charge, voltage, and capacitance
    - Charging and discharging of a capacitor
  - Relationship between voltage, current, and capacitance; power; and energy
  - Equivalent capacitance when a set of capacitors are in series and in parallel

Chapter 6.1 in Basic Engineering Circuit Analysis by Irwin and Nelms
Capacitors

- Composed of two conductive plates separated by an insulator (or dielectric).
- Commonly illustrated as two parallel metal plates separated by a distance, d.

\[ C = \varepsilon \frac{A}{d} \]

where \( \varepsilon = \varepsilon_r \varepsilon_0 \)

\( \varepsilon_r \) is the relative dielectric constant
\( \varepsilon_0 \) is the vacuum permittivity
Effect of Dimensions

- Capacitance increases with
  - increasing surface area of the plates,
  - decreasing spacing between plates, and
  - increasing the relative dielectric constant of the insulator between the two plates.
Types of Capacitors

- Fixed Capacitors
  - Nonpolarized
    - May be connected into circuit with either terminal of capacitor connected to the high voltage side of the circuit.
      - Insulator: Paper, Mica, Ceramic, Polymer
  - Electrolytic
    - The negative terminal must always be at a lower voltage than the positive terminal
      - Plates or Electrodes: Aluminum, Tantalum
Nonpolarized

- Difficult to make nonpolarized capacitors that store a large amount of charge or operate at high voltages.
- Tolerance on capacitance values is very large
  - +50%/-25% is not unusual

http://www.marvac.com/fun/ceramic_capacitor_codes.aspx
Electrolytic

Pspice Symbols

Fabrication

Variable Capacitors

- Cross-sectional area is changed as one set of plates are rotated with respect to the other.

MEMS Capacitor

- MEMS (Microelectromechanical system)
  - Can be a variable capacitor by changing the distance between electrodes.
  - Use in sensing applications as well as in RF electronics.

Electric Double Layer Capacitor

- Also known as a supercapacitor or ultracapacitor
  - Used in high voltage/high current applications.
    - Energy storage for alternate energy systems.

Electrical Properties of a Capacitor

- Acts like an open circuit at steady state when connected to a d.c. voltage or current source.
- Voltage on a capacitor must be continuous
  - There are no abrupt changes to the voltage, but there may be discontinuities in the current.
- An ideal capacitor does not dissipate energy, it takes power when storing energy and returns it when discharging.
Properties of a Real Capacitor

- A real capacitor does dissipate energy due leakage of charge through its insulator.
- This is modeled by putting a resistor in parallel with an ideal capacitor.
Energy Storage

- Charge is stored on the plates of the capacitor.

Equation:

\[ Q = CV \]

Units:

Farad = Coulomb/Voltage
Farad is abbreviated as F
Sign Conventions

- The sign convention used with a capacitor is the same as for a power dissipating device.
  - When current flows into the positive side of the voltage across the capacitor, it is positive and the capacitor is dissipating power.
  - When the capacitor releases energy back into the circuit, the sign of the current will be negative.
Charging a Capacitor

- At first, it is easy to store charge in the capacitor.
- As more charge is stored on the plates of the capacitor, it becomes increasingly difficult to place additional charge on the plates.
  - Coulombic repulsion from the charge already on the plates creates an opposing force to limit the addition of more charge on the plates.
    - Voltage across a capacitor increases rapidly as charge is moved onto the plates when the initial amount of charge on the capacitor is small.
    - Voltage across the capacitor increases more slowly as it becomes difficult to add extra charge to the plates.
Adding Charge to Capacitor

The ability to add charge to a capacitor depends on:
- the amount of charge already on the plates of the capacitor
- the force (voltage) driving the charge towards the plates (i.e., current)
Discharging a Capacitor

• At first, it is easy to remove charge in the capacitor.
  • Coulombic repulsion from charge already on the plates creates a force that pushes some of the charge out of the capacitor once the force (voltage) that placed the charge in the capacitor is removed (or decreased).

• As more charge is removed from the plates of the capacitor, it becomes increasingly difficult to get rid of the small amount of charge remaining on the plates.
  • Coulombic repulsion decreases as charge spreads out on the plates. As the amount of charge decreases, the force needed to drive the charge off of the plates decreases.
    • Voltage across a capacitor decreases rapidly as charge is removed from the plates when the initial amount of charge on the capacitor is small.
    • Voltage across the capacitor decreases more slowly as it becomes difficult to force the remaining charge out of the capacitor.
Current-Voltage Relationships

\[ q = C v_c \]

\[ i_c = \frac{dq}{dt} \]

\[ i_c = C \frac{dv_c}{dt} \]

\[ v_c = \frac{1}{C} \int_{t_0}^{t_1} i_c \, dt \]
Power and Energy

\[ p_C = i_C v_C \]

\[ p_C = C v_C \frac{d v_C}{d t} \]

\[ w_C = \frac{1}{2} C v_C^2 \]

\[ w_C = \frac{q^2}{2C} \]
Capacitors in Parallel
\( C_{\text{eq}} \) for Capacitors in Parallel

\[ i_{in} = i_1 + i_2 + i_3 + i_4 \]

\[ i_1 = C_1 \frac{dv}{dt} \]

\[ i_2 = C_2 \frac{dv}{dt} \]

\[ i_3 = C_3 \frac{dv}{dt} \]

\[ i_4 = C_4 \frac{dv}{dt} \]

\[ i_{in} = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + C_4 \frac{dv}{dt} \]

\[ i_{in} = C_{eq} \frac{dv}{dt} \]

\[ C_{eq} = C_1 + C_2 + C_3 + C_4 \]
Capacitors in Series
**C_{eq} for Capacitors in Series**

\[ v_{in} = v_1 + v_2 + v_3 + v_4 \]

\[ v_1 = \frac{1}{C_1} \int_{t_0}^{t_1} idt \]
\[ v_2 = \frac{1}{C_2} \int_{t_0}^{t_1} idt \]
\[ v_3 = \frac{1}{C_3} \int_{t_0}^{t_1} idt \]
\[ v_4 = \frac{1}{C_4} \int_{t_0}^{t_1} idt \]

\[ v_{in} = \frac{1}{C_1} \int_{t_0}^{t_1} idt + \frac{1}{C_2} \int_{t_0}^{t_1} idt + \frac{1}{C_3} \int_{t_0}^{t_1} idt + \frac{1}{C_4} \int_{t_0}^{t_1} idt \]

\[ v_{in} = \frac{1}{C_{eq}} \int_{t_0}^{t_1} idt \]

**C_{eq} = \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \right]^{-1}**
General Equations for $C_{eq}$

**Parallel Combination**

- If $P$ capacitors are in parallel, then

$$C_{eq} = \sum_{p=1}^{P} C_p$$

**Series Combination**

- If $S$ capacitors are in series, then:

$$C_{eq} = \left[ \sum_{s=1}^{S} \frac{1}{C_s} \right]^{-1}$$
Summary

- Capacitors are energy storage devices.
- An ideal capacitor acts like an open circuit at steady state when a DC voltage or current has been applied.
- The voltage across a capacitor must be a continuous function; the current flowing through a capacitor can be discontinuous.
- The equations for equivalent capacitance for

  - Capacitors in parallel
    \[ C_{eq} = \sum_{p=1}^{P} C_p \]

  - Capacitors in series
    \[ C_{eq} = \left[ \sum_{s=1}^{S} \frac{1}{C_s} \right]^{-1} \]