Student’s Name: ______________________________________

Instructions:

1. This is a closed book/closed notes exam. Scrap paper has been provided in this file for some of the problems. You may add extra pages if needed.

2. A calculator, which may be a graphing calculator or a calculator (not MATLAB or Mathematica) on a computer may be used to perform the required calculations.

3. A computer may be used during the exam to download the test, to write answers on the exam electronically, to submit answers on Scholar, and to upload a file containing the work that you performed when calculating the answers to the problems.

4. Formatting answers:
   a. Use proper units and prefixes
   b. Use the passive sign convention
   c. Final answers should be expressed using three significant figures

5. Show all work.
   a. Full credit will not be given for correct answers without work that supports the answers.
   b. Partial credit will only be given if sufficient work is shown so that errors in calculations or conceptual errors can be traced.
1. For the circuit below:

![Circuit Diagram]

a. Write an equation for the power dissipated in R2 using the variable \( I_{in} \), \( R_1 \), and \( R_2 \).

\[
I_{R2} = \frac{R_1}{R_1 + R_2} I_{in}
\]

\[
V_{R2} = I_{in} \frac{R_1 R_2}{R_1 + R_2}
\]

\[
P_{R2} = V_{R2} I_{R2} = \left(\frac{I_{in}^2}{R_2}\right) R_2 = \frac{V_{R2}^2}{R_2}
\]

\[
P_{R2} = \frac{R_1^2 R_2}{(R_1 + R_2)^2} I_{in}^2
\]

b. Given that \( R_1 \) is not zero or infinite ohms, show mathematically that the power dissipated in R2 is a maximum when \( R_2 = R_1 \).

This is the maximum power transfer theorem.

\[
P_{R2} = \frac{R_1^2 R_2}{(R_1 + R_2)^2} I_{in}^2
\]

Clearly, \( P_{R2} = 0 \) W when \( R_2 = 0 \Omega \). When \( R_2 = \infty \Omega \), \( P_{R2} = \frac{R_1^2 R_2}{(R_1 + R_2)^2} I_{in}^2 \approx \frac{1}{\infty} W = 0W
\]

Ideally, the students would have taken the first derivative of the equation with respect to \( R_2 \) to find the point where the slope was equal to zero and then take the second derivation to show that the point was a maximum.

\[
\frac{dP_{R2}}{dR2} = \frac{R_1^2}{(R_1 + R_2)^2} I_{in}^2 - 2 \frac{R_1 R_2}{(R_1 + R_2)^3} I_{in}^2
\]

\[
\frac{dP_{R2}}{dR2}_{R_1=R_2} = \frac{R_2^2}{(R_2 + R_2)^3} I_{in}^2 - 2 \frac{R_2^2}{(R_2 + R_2)^3} I_{in}^2
\]

\[
\frac{dP_{R2}}{dR2}_{R_1=R_2} = \frac{1}{4} I_{in}^2 - 2 \left(\frac{1}{8}\right) I_{in}^2 = 0
\]
\[
\frac{d^2 P_{R_2}}{dR_2^2} = \left( -2 \frac{R_1}{(R_1 + R_2)^3} I_{in}^2 \right) + \left[ -2 \frac{R_1^2}{(R_1 + R_2)^3} I_{in}^2 + \frac{6R_1 R_2}{(R_1 + R_2)^4} I_{in}^2 \right]
\]

\[
\frac{d^2 P_{R_2}}{dR_2^2} \bigg|_{R_1 = R_2} = \left( -2 \frac{R_2^2}{(R_2 + R_2)^3} I_{in}^2 \right) + \left[ -2 \frac{R_2}{(R_2 + R_2)^3} I_{in}^2 + \frac{6R_2^2}{(R_2 + R_2)^4} I_{in}^2 \right]
\]

\[
\frac{d^2 P_{R_2}}{dR_2^2} \bigg|_{R_1 = R_2} = \frac{-2}{8R_2} I_{in}^2 + \frac{-2}{8R_2} I_{in}^2 + \frac{6}{16R_2} I_{in}^2 = -\frac{2}{8R_2} I_{in}^2
\]

Since the second derivative is negative, the equation has a maxima at \( R_1 = R_2 \).

A graph of the equation is also sufficient.

The maximum power dissipated through \( R_2 \) occurs when \( R_2 = R_1 \) and the maximum power is \( 0.25(I_{in}^2)R_2 \).

c. What should the value of \( R_2 \) be if you wanted the voltage across \( R_2 \) to be a maximum?

\( R_2 \) should be equal to \( \infty \) \( \Omega \) as the maximum voltage across \( R_2 \) is \( I_{in}R_1 \).

d. What should the value of \( R_2 \) be if you wanted the current flowing through \( R_2 \) to be a maximum?

\( R_2 \) should be equal to \( 0 \) \( \Omega \) as the maximum current through \( R_2 \) is \( I_{in} \).

NOTE: If \( R_1 \) is allowed to be \( \infty \) \( \Omega \), then \( R_2 \) can be any value less than \( \infty \) \( \Omega \).
2. The assumptions that are used when a feedback resistor, \( R_f \), connects the negative input terminal to the output terminal of an ideal operational amplifier are:
   - \( v_d = v_2 - v_1 = 0 \) V
   - no current flows into or out of the input terminals
   - the internal gain of the operational amplifier is infinite
   - the output of the operational amplifier can supply or sink as much current as necessary

a. Redraw the circuit above with the model used for the ideal operational amplifier.
b. Explain how the model incorporates each of the assumptions described above.

There is a connection between the two inputs of the operation amplifier, Rin. However, the resistance of Rin = ∞ Ω. This means that no current can flow between two input terminals (second bullet).

Since the current through Rin is equal to 0 A, the voltage drop across Rin must be 0 V. This means that there is no difference between the voltage at the positive input and the negative input terminals (first bullet).

The gain of the op amp, A, is the voltage gain of the dependent voltage source. A is equal to ∞ for an ideal operational amplifier (third bullet).

The output resistor of the operational amplifier must be equal to 0 Ω. When this is true, the only components that determine the amount of current that flows into or out of the output terminal of the operational amplifier are V1, R1, Rf, and RL, all components that are external to the operational amplifier (fourth bullet).

c. Select one assumption and explain how the operation of a nonideal operational amplifier differs from the ideal operational amplifier. For example, pick a component in the model for the ideal operational amplifier and state how its value should change to model a nonideal operational amplifier. Or, identify a component that should be added to the ideal operational amplifier model to alter the model such that the model no longer meets the assumption that you selected.

Bullet 1 and 2: Rin is not equal to ∞ Ω.

There will be current flowing in one terminal and out of the other terminal of the operational amplifier. This will reduce the amount of current flowing through Rf, which will reduce the voltage that must be outputted by the operational amplifier. I.e., Vo(nonideal) is smaller than Vo(ideal).

Bullet 3 and 4: If A is not equal to ∞.

The maximum output voltage of the operational amplifier will be limited by the gain. In turn, this will limit the maximum current that can flow into or out of the operational amplifier.

Bullet 4: If Ro is not equal to 0 Ω.

The maximum current that flows out of or into the operational amplifier is no longer infinite, but is equal to Av_d/Ro.

Additional components that are added to the model are V+ and V-, the power supplies for the operational amplifier. The magnitude of these power supplies limit the maximum and minimum voltages that can be outputted by the dependent current source, which then limits the current that can flow into or out of the output terminal of the operational amplifier.
3. For the circuit below, no calculations are required.
   a. Redraw the subcircuits that should be analyzed when applying superposition.
   b. In each subcircuit, identify the resistors that you know have no current flowing through them without having to perform any calculations if there are any in that subcircuit.

There are three subcircuits that should be drawn. V1 and V2 will be replaced by short circuits or the magnitude of each source should be set to 0 V, when they are turned off. I1 will be replaced by an open circuit or the magnitude should be set to 0 A. The dependent voltage source, 5 Vx, should remain in all three subcircuits.

The resistors that are circled in red are the ones that have no current flowing through them.
Or the sources can remain, but the magnitudes of the sources set to zero.
4. For the following circuit,

\[ \frac{V_o}{V_{in}} = -\left[1 + \frac{R_f}{R_1}\right] \]

a. Write an equation for the voltage transfer characteristic \((V_o/V_{in})\) where you assume that the operational amplifier is ideal.

b. Instead, an almost ideal amplifier is used in the circuit on the previous page, where:

- \(V^+ = 12 \, \text{V}\) and \(V^- = -9 \, \text{V}\)
- \(R_1 = 5 \, \text{k}\Omega\), \(R_f = 30 \, \text{k}\Omega\), and \(R_L = 50 \, \text{k}\Omega\)

i. Calculate the value of the input voltage such that the operational amplifier circuit just enters the positive saturation region.

\[ V_o = -\left[1 + \frac{R_f}{R_1}\right] V_{in} \]
\[ 12V = -\left[1 + \frac{30\text{k}\Omega}{5\text{k}\Omega}\right] V_{in} \]
\[ V_{in} = -\frac{12}{7} V = -1.71V \]

ii. Calculate the value of the input voltage such that the operational amplifier circuit just enters the negative saturation region.

\[ V_o = -\left[1 + \frac{R_f}{R_1}\right] V_{in} \]
\[ -9V = -\left[1 + \frac{30\text{k}\Omega}{5\text{k}\Omega}\right] V_{in} \]
\[ V_{in} = \frac{9}{7} V = -1.29V \]
5. For the circuit below where the load resistor is RL:

a. Determine the Thévenin equivalent circuit and the Norton equivalent circuit.

There are several different ways to find the Thévenin equivalent resistor.

Method 1:

a. Turn off all of the current and voltage sources and combine resistors until there is only one in parallel or in series with the load resistor, RL.

b. Replace RL with an open circuit. The voltage across the open circuit, \( V_{oc} \), is the Thévenin voltage, \( V_{Th} \).

\[ V_{Th} = 2 \, V \]

c. Replace RL with a short circuit. The current flowing through the short circuit, \( I_{sc} \), is the Norton current, \( I_N \).

\[ I_N = 0.833 \, mA \]
Note that these last two circuits must be drawn, no matter what method is used to find \( V_{Th} \) and \( I_{N} \).
6. Solve for $I_x$ and $V_x$ using any of the analysis techniques that you have learned thus far.

Whatever way is used to find the values for $I_x$ and $V_x$, the answers should be:

$I_x = 74.3$ mA

$V_x = 7.31$ V