Additional notes on homography

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1 Homography matrix

Given a set of corresponding image points P and Q in two images, we want to estimate the 3×3 homography H, such that $q \equiv Hp$, where p represents the homogeneous coordinate of P and q represents the homogeneous coordinate of Q. Let (u_q, v_q) and (u_p, v_p) represent the actual image coordinates of Q and P respectively. The homography equation can then be rewritten as

$$\begin{bmatrix} wu_q \\ wv_q \\ w \end{bmatrix} = H \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

This equation can be solved as below:

$$u_q = \frac{h_1^T p}{h_3^T p}, v_q = \frac{h_2^T p}{h_3^T p}, where\ H = \begin{bmatrix} h_1^T \\ h_2^T \\ h_3^T \end{bmatrix}$$

Converting the above equations in linear form,

$$h_1^T p - u_q(h_3^T p) = 0, h_2^T p - v_q(h_3^T p) = 0$$

Given n corresponding pairs of points (p_i, q_i), we can combine all the equations in a matrix form as $Lh = 0$, where

$$L = \begin{bmatrix} p_1^T & 0 & -u_{1q} p_1^T \\ 0 & p_1^T & -v_{1q} p_1^T \\ ... & & \\ p_n^T & 0 & -u_{nq} p_n^T \\ 0 & p_n^T & -v_{nq} p_n^T \end{bmatrix}$$

and

$$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

Since H is defined only up to scale, we put additional constraint on H, *i.e.*, $||h||^2 = 1$. The least squares solution of h is then given by the eigenvector corresponding to the smallest eigenvalue of matrix $L^T L$.