1. Derive the z-transform and associated region of convergence for the following signals using the definition
   
   (a) \( x[n] = u[n] \)
   
   (b) \( x[n] = \frac{1}{2} u[n] \)

2. Find the z-transform for the following causal signals using the table and properties
   
   (a) \( \frac{1}{3} u[n] \)
   
   (b) \( u[n] + \frac{1}{3} u[n - 3] \)
   
   (c) \( \frac{1}{2} u[n] + \frac{1}{2} \cos(\pi n/4) u[n] \)

3. Find the discrete-time transfer function for the following systems described by their continuous-time transfer function
   
   (a) \( H(s) = \frac{1}{s + 2} \)
   
   (b) \( H(s) = \frac{1}{s^2 + 2s + 1} \)
   
   (c) \( H(s) = \frac{s}{s^2 + 2s + 1} \)

4. Find the inverse z-transform for the following causal signals using the table and properties
   
   (a) \( H(z) = \frac{6}{1 - 0.25z^{-1}} \)
   
   (b) \( H(z) = \frac{1}{(1 - 0.5z^{-1})(1 - 0.75z^{-1})} \)
   
   (c) \( H(z) = \frac{1 + z^{-1}}{(1 - 0.5z^{-1})(1 - 0.75z^{-1})} \)

5. Solve the following difference equations using the z-transform, separating the zero-input and zero-state responses
(a) \[2y[n + 1] + y[n] = x[n] + x[n - 1]\] where \(y[0] = 1, y[1] = 1\) and \(x[n] = u[n]\)

(b) \[2y[n] + y[n - 1] = x[n]\] where \(y[-1] = 1\) and \(x[n] = \cos(\pi n/4)u[n]\)

(c) \[5y[n + 2] + 4y[n + 1] + 3y[n] = 2x[n]\] where \(y[-2] = 1, y[-1] = 2\) and \(x[n] = u[n]\)

6. Determine the stability of the following systems and implement each in canonical (Direct Form II) form using unit delays.

(a) \[H(z) = \frac{2z^2 - 3z + 1}{z^2 - 2z + 10}\]

(b) \[H(z) = \frac{2z^3 + z^2 + 5z + 1}{z^3}\]

7. For each of your implementations of the previous problem, write the system as a state equation.

8. Find the magnitude and phase response of the following systems

(a) \[y[n + 1] + \frac{1}{6}y[n] = \frac{1}{2}x[n]\]

(b) \[H(z) = \frac{12z - 2}{4z - 1}\]

9. For each system in the previous problem, what is the steady-state output for the input \(x[n] = \cos(\pi n/2)u[n]\)?

10. Given the following CT transfer function implementing a third-order low-pass filter

\[H(s) = \frac{K}{(s + \omega_c)(s^2 + \sqrt{2}\omega_c s + \omega_c^2)}\]

(a) Design a discrete-time system to approximate the system

(b) Implement your design in canonical form

(c) Choose an appropriate sampling interval relative to \(\omega_c\) with a justification