1. For each of the following signals sampled at the indicated rate, sketch the frequency spectrum before and after sampling. Indicate if the signal can be reconstructed exactly using an ideal low-pass filter

(a) \( x(t) = 4 + 8 \cos(8\pi t) \) sampled at 16 Hz.
(b) \( x(t) = 4 + 8 \cos(8\pi t) \) sampled at 6 Hz.
(c) \( x(t) = 1 + 5 \cos(10\pi t) + 2 \cos(15\pi t) + 9 \sin(16\pi t) \) sampled at 30 Hz.

2. Suppose the signal having the frequency spectrum below is sampled.

![Frequency spectrum](image)

(a) what is the Nyquist sampling rate for the signal?
(b) plot the frequency spectrum after sampling for the rate determined in a)
(c) plot the frequency spectrum after sampling for half and twice the rate determined in a)

3. For the following DT signals, determine the energy and power, and classify them as energy signals, power signals, or neither

(a) \( x[n] = 8^{-n}u[n] \)
(b) \( x[n] = 8^n u[n] \)
(c) \( x[n] = \begin{cases} 
0 & n = 0 \\
1 & |n| \text{ odd} \\
-1 & |n| \text{ even}
\end{cases} \)
(d) \( x[n] = u[-n + 5] + u[n - 5] \)

4. For each of the following values of \( \gamma \) sketch the signal \( x[n] = \gamma^n u[n] \).

(a) \( \gamma = \frac{1}{2} \)
(b) \( \gamma = -\frac{1}{2} \)
(c) \( \gamma = 2 \)
(d) \( \gamma = -2 \)
(e) \( \gamma = \frac{1}{2} + j\frac{1}{2} \)

5. Given the following signals, sketch each as well as the time reversed and left/right shifted versions

(a) \( x[n] = u[n - 1] - u[n - 5] \)
(b) \( x[n] = (n + 4)u[-n] + (-n + 4)u[n] \)
(c) \( x[n] = \delta[n - 1] + 2\delta[n - 3] \)
(d) \( x[n] = \delta[n + 1] + 2\delta[n + 3] \)
(e) \( x[n] = \cos(3\pi n + \pi/4) \)
(f) \( x[n] = \frac{1^n}{2} + \frac{1^n}{3} \)
(g) \( x[n] = 2^n u[n] \)

6. Given the following signals, \( x[n] \) and \( y[n] \), write each as a combination of the signal models \( \delta[n] \), \( u[n] \), \( \gamma^n \)

7. Classify the following systems (written as input \( \mapsto \) output) according to linearity, time-invariance, causality, stability (to the extent possible), and memory

(a) \( x[n] = u[n] \mapsto y[n] = 4^{-n} u[n] \)
(b) \( x[n] = \delta[n] \mapsto y[n + 1] = x[n] + y[n] \)
(c) $x[n] = u[-n] \mapsto y[n] = 10^n u[n]$
(d) $x[n] = n \mapsto y[n] = 3n^2 + 1$
(e) $x[n] = 12^n u[n] \mapsto y[n] = x[n + 1]$

8. Consider a system that, when an impulse is applied, generates the Fibonacci sequence as its output starting at $n = 0$.

(a) describe this system by a difference equation
(b) is the system stable?

9. Repeat the previous problem but assume a step input is applied instead.

10. Solve for the zero-input response given the following systems

(a) $2y[n + 1] + y[n] = x[n] + x[n - 1]$ where $y[0] = 1$
(b) $2y[n] + y[n - 1] = x[n]$ where $y[-1] = 1$
(c) $5y[n + 2] + 4y[n + 1] + 3y[n] = 2x[n]$ where $y[-2] = 1, y[-1] = 2$

11. Determine the impulse response from the following difference equations in closed form and iteratively for $n = \{0, 1, 2\}$

(a) $y[n + 1] + \frac{1}{4} y[n] = x[n]$
(b) $y[n + 1] + 2y[n] = 5x[n - 1]$
(c) $y[n + 2] + \frac{1}{2} y[n + 1] = x[n]$
(d) $y[n + 2] + y[n + 1] + \frac{5}{12} y[n] = x[n]$
(c) $y[n] + \frac{1}{2} y[n - 1] + \frac{5}{8} y[n - 2] = 2x[n - 1]$

12. Using the definition of convolution find a closed form solution to $x_1[n] \ast x_2[n]$ where $x_1[n] = u[n]$ and $x_2[n] = \left(\frac{1}{2}\right)^n u[n]$

13. For the impulse responses and inputs below determine the zero-state response using a convolution table

(a) $h[n] = \left(\frac{1}{2}\right)^n u[n]$ and $x[n] = u[n]$
(b) $h[n] = \left(\frac{1}{2}\right)^n u[n]$ and $x[n] = u[n] - u[n - 5]$
(c) $h[n] = 2\delta[n] + (2)^n u[n]$ and $x[n] = \cos(4\pi n + \pi/8) u[n]$
14. Determine $x_1[n] * x_2[n]$ for the following signals using a graphical method

\[ x_1[n] = \{ \cdots, 0, 1, 2, 1, 3, 0, \cdots \} \]
\[ x_2[n] = \{ \cdots, 0, 1, 2, 3, 2, 1, 0, \cdots \} \]

15. Write a MATLAB function, convolution.m, that accepts two finite-length signals as a pair of indices, values and returns the result of convolving them in the same form. Your function should produce an error if inconsistent arguments are passed to it. Use the following function signature:

\[ [n, c] = \text{convolution}(n1, x1, n2, x2) \]

Check your answer to the previous question using your function.

16. Determine the external (BIBO stability) from the system impulse responses in Problem 13 a)-c).

17. Determine the internal stability for each difference equation system model in Problem 11 a)-e).