Today we will see two applications of the z-transform. After today you should be able to:

1. use the z-transform to solve linear difference equations
2. implement a transfer function using unit delays

Solving Difference Equations

Recall the Laplace transform can be used to transform a linear differential equation into an algebraic equation, easing their solution. The z-transform can be used similarly to solve difference equations.

We can use either the left or right shift theorem as the basis of the conversion. Suppose \( x[n]u[n] \leftrightarrow X(z) \):

- **Right-Shift Theorem** \((m > 0)\)
  \[
x[n - m]u[n] \leftrightarrow \frac{1}{z^m}X(z) + \frac{1}{z^m} \sum_{n=1}^{m} x[-n]z^n
  \]

- **Left-Shift Theorem** \((m > 0)\)
  \[
x[n + m]u[n] \leftrightarrow z^mX(z) - z^m \sum_{n=0}^{m-1} x[n]z^{-n}
  \]

Note the right-shift uses initial conditions, while the left-shift uses auxiliary conditions. Even though we can convert between them by running the difference equation forward or backward, it’s generally easier to just use the form that corresponds to the conditions given and convert to/from the advanced/delayed form of the difference equation. We will see examples in class.

Implementing Transfer Functions

We can implement a discrete transfer function using the same techniques as for continuous time, however the basic building block is the unit delay \((z^{-1})\) rather than an integrator.

We will see several examples, one of which is the Finite Impulse Response (FIR) filter. FIR filters have a finite number of non-zero terms in their impulse response and are easily implemented as a linear combination of delayed inputs.