This problem set covers Fourier methods – Chapters 6 and 7 of the text and Lectures 30-37.

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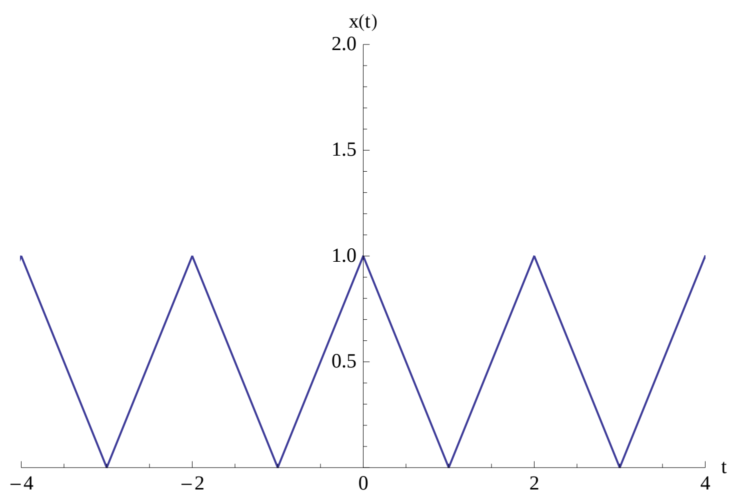
1. [C] For the periodic signal

$$x(t) = (t+1)^2 \text{ for } -1 < t < 1 \text{ extended periodically with period } T_0 = 2$$

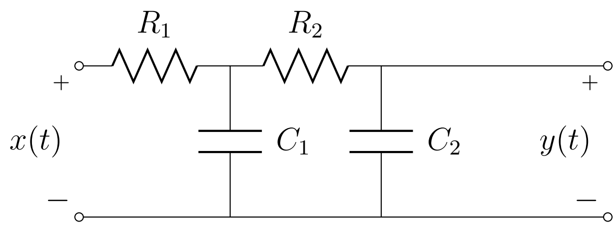
   (a) Determine its Fourier Series using the trig form
   (b) Determine its Fourier Series using the compact trig form and your result from part a
   (c) Plot the spectrum using your results from part b
   (d) Plot the original signal and the Fourier series approximation over one period for the first three non-zero terms.

2. Obtain the exponential form of the Fourier series expansions for each of the signals below (if it exists) without using integration, but applying trigonometric identities and Euler's formula. Be sure to state what the fundamental period is.
   (a) $\cos(\frac{3\pi}{2}t)$
   (b) $\sin(2t)$
   (c) $\cos(5\pi t) + \sin(7\pi t + \pi/9)$
   (d) $\sin(10\pi t) + 2\cos(10t)$

3. [C] Given the periodic signal below



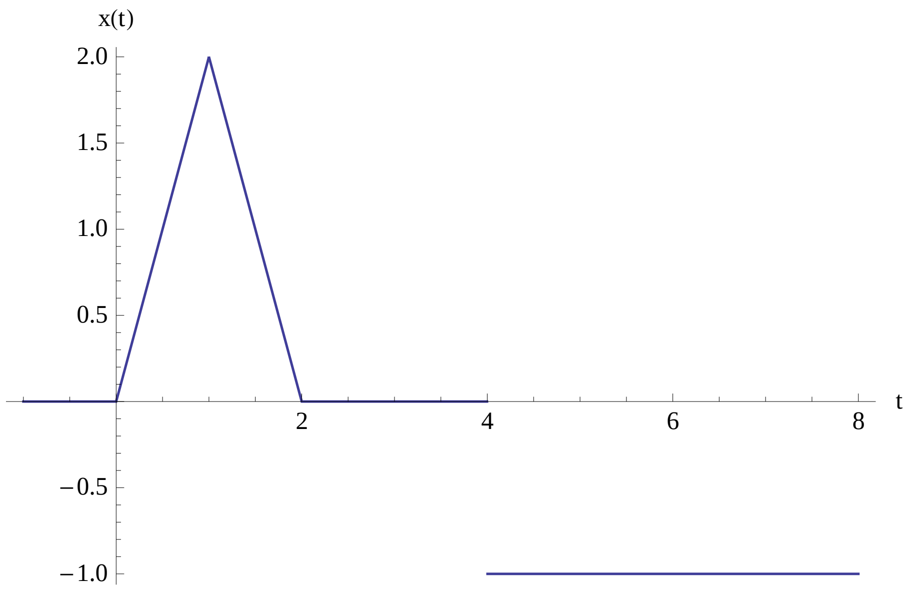
   (a) Determine the Fourier series spectrum for the signal.
   (b) If the signal is applied to the input of the following circuit where $R_1 = R_2 = 10\,\text{k}\Omega$, and $C_1 = C_2 = 1\mu\text{F}$,



what would be the output response be?

(c) Plot the input from part a and the response from part b in the time domain on the same plot using $N = 10$ terms of the Fourier series.

4. Given the following signal from problem set 2



(a) C determine the Fourier Transform using the definition (hint: write the signal as a piece-wise function and use the same limit process as we did in class for the Fourier transform of the unit step),

(b) In PS2 we could write the signal as a sum of simple signal models. Then using the table of known transforms and the transform properties, we could determine the Laplace Transform. Can we use the same approach to find the Fourier transform here? Why or Why not?

5. Use the definition of the Fourier Transform, the table of known transforms, and/or the transform properties to determine the transform of the following time-domain signals:

(a) $x_1(t) = e^{-t^2}$

(b) $x_2(t) = x_1(t-2) + x_1(t+2)$

(c) $x(t) = x_2'(t)$

(d) $x(t) = x_1(t)\cos(100t)$

6. Use the definition of the inverse Fourier Transform, the table of known transforms, and/or the transform properties to determine the inverse transform of the following frequency-domain signal:

$$X(\omega) = e^{-(\omega-5)^2} + e^{-(\omega+5)^2}$$

7. [C] Consider a model of a signal that is a sinusoidal function turned on at $t = -\tau/2$ and off at $t = \tau/2$:

$$x(t) = \sqcap(t/\tau)\cos(\omega_0 t)$$

where $\sqcap(t)$ is the gate function

$$\sqcap(t) = \begin{cases} 1 & |t| < 1 \\ 0 & \text{else} \end{cases}$$

(a) Find the Fourier Transform of $x(t)$ as a function of ω, ω_0, and τ.

(b) Suppose this signal is applied to a stable LTIC system. How could you easily approximate the response of the system to $x(t)$?

(c) For your approximation in part b) for what values of ω_0, and τ would the approximation be most correct?

(d) For your approximation in part b) for what time values would the approximation be most correct?

8. [C] Suppose a system has a transfer function

$$H(s) = \frac{4}{s^2 + 5s + 4}$$

(a) Using Fourier analysis, what is the zero-state response in the Fourier domain if the input is $x(t) = u(t)$

(b) Using your result from part a) what is the zero-state response in the time domain?

(c) Using Fourier analysis, what is the zero-state response in the Fourier domain if the input is $x(t) = \sqcap(t)$

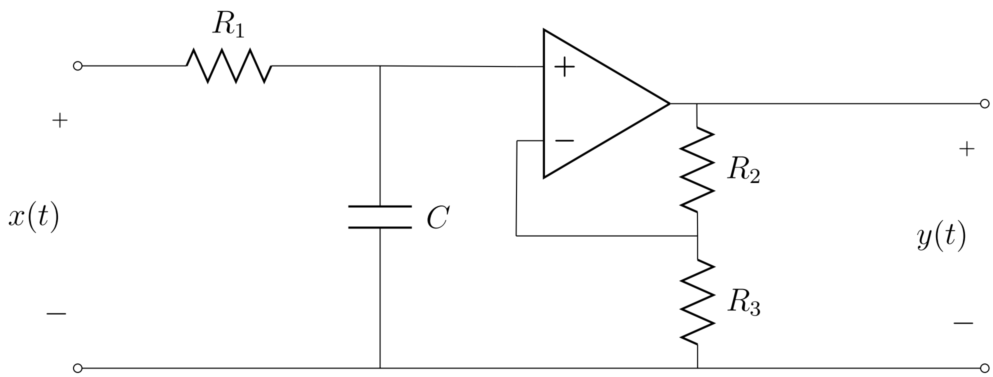
(d) Using your result from part c) what is the zero-state response in the time domain?

(e) Compare the solution to parts a-d (using Fourier methods) to how you would have solved it using Laplace. How are the methods similar/different?

9. [L] [C] Consider the transfer function of a first-order low-pass filter from problem set 2

$$H(s) = \frac{K}{s + a}$$

for constants $K > 0$ and $a > 0$. The solutions provided a single op-amp circuit to implement this filter



(a) Choose values for the components so that $a = 1000$ and the DC gain is 0 dB.

(b) Plot the frequency response from part a as a Bode plot.

(c) Build the circuit (using supply rails of +/- 5V) and experimentally measure the frequency response (gain and phase shift) by applying a 2V P-P sinusoid as the input at the following frequencies: 10 Hz, 100 Hz, 500 Hz, 1 kHz, 2 kHz, 5 kHz, 10 kHz.

(d) Compare your measured results to those of parts a and b by plotting your experimental results on top of your Bode plot from part b.