This problem set covers the complex, frequency domain methods – Chapters 4 of the text and Lectures 16-27.

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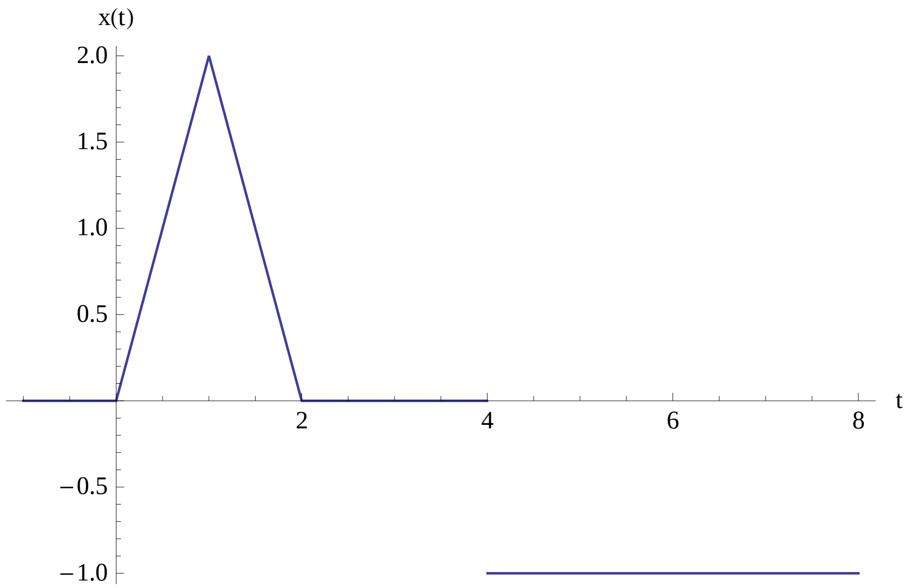
1. Given the following signal



   (a) determine the Laplace Transform, including the region-of-convergence, using the definition,
   (b) write the signal as a sum of simple signal models, and
   (c) using the table of known transforms, and/or the transform properties to determine the Laplace Transform of part b).

2. Use the definition of the Laplace Transform, the table of known transforms, and/or the transform properties to determine the transform of the following functions including their region-of-convergence:
   (a) $x_1(t) = tu(t) - tu(t-2) + 2u(t-2)$
   (b) $x_2(t) = x_1(10t)$
   (c) $x(t) = x_1'(t) + \int_{0^-}^{t} x_2(\tau)\, d\tau$
   (d) $x(t) = te^{-t}\cos(2t)u(t)$

3. Using the table of known transforms and the transform properties to determine the Laplace transform of the following signals, expressing each as a single ratio of polynomials:
   (a) $x(t) = 2\delta(t) + te^{-t}u(t) + 4e^{-2t}u(t)$
   (b) $x(t) = (1 - e^{-4t})u(t)$
   (c) $x(t) = e^{-3t}(\cos(2\pi t) + \sin(2\pi t))u(t)$
   (d) $x(t) = 10e^{-5t}\cos(\pi t + \pi/4)u(t)$
   (e) $x(t) = (e^{-10t}(1 + 2t + 5t^2)u(t)$

4. Using the table of known transforms and the transform properties to determine the inverse Laplace transform assuming all are causal signals:
   (a) $X(s) = \frac{1}{s^2+5s+4}$
   (b) $X(s) = \frac{1}{s^3+9s^2+24s+16}$
   (c) $X(s) = \frac{5}{s^3+s^2+25s+25}$
   (d) $X(s) = \frac{s}{s^3+s^2+9s+9}$
   (e) $X(s) = \frac{s+3}{s^2+6s+45}$
   (f) $X(s) = \frac{6}{s^2+6s+45}$
   (g) $X(s) = \frac{s+6}{s^2+2s+26}$

5. Given a system described by the following differential equation

$$y'' + 9y' + 20y = x$$

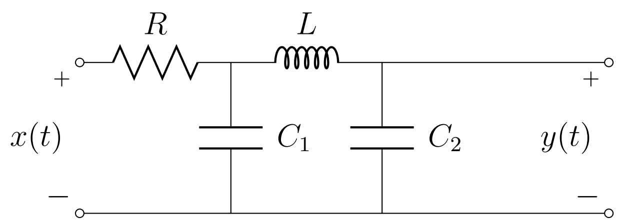
   where $y'(0^-) = 0$ and $y(0^-) = 2$.
   (a) find the zero-input response in the Laplace domain
   (b) determine the Transfer Function of the system
   (c) determine the zero-state response for $x(t) = e^{-t}u(t)$ in the Laplace domain.

6. Given a system described by the following differential equation

$$y'' + 2y' + 10y = x' + 3x$$

   where $y'(0^-) = 0$ and $y(0^-) = 0$.
   (a) determine the Transfer Function of the system
   (b) determine the total response when $x(t) = (e^{-2t} + 2e^{-5t})u(t)$ in the time domain.

7. Given the circuit from problem set 2,
   (a) transform the circuit into the Laplace domain assuming zero initial conditions
   (b) determine the Transfer Function of the system in terms of R, C_1, C_2 and L
   (c) show the result in part b is the same as the Laplace Transform of the impulse response from PS2.



8. Consider the transfer function of a first-order low-pass filter

$$H(s) = \frac{K}{s+a}$$

for constants $K > 0$ and $a > 0$.
   (a) implement the transfer function as a block diagram in canonical form (Direct-Form II), in term of integrators, multipliers, and sums.
   (b) draw a circuit that implements part a and thus the transfer function in terms of op-amps, resistors, and capacitors.