① a)

$$x(t)=\begin{cases} 0 & t<0 \\ 2t & 0<t<1 \\ 4-2t & 1<t<2 \\ 0 & 2<t<4 \\ -1 & t>4 \end{cases}$$

$$X(s)=\int_{-\infty}^{\infty} x(t)e^{-st}dt=\int_0^1 2te^{-st}dt+\int_1^2(4-2t)e^{-st}dt+\int_4^{\infty}-e^{-st}dt$$

$$=\frac{-2}{s^2}(1+st)e^{-st}\Big|_0^1+\frac{2}{s^2}(1+(t-2)e^{-st})\Big|_1^2+\frac{1}{s}e^{-st}\Big|_4^{\infty}$$

$$=\frac{-2}{s^2}\left(1-(s+1)e^{-s}\right)+\frac{2}{s^2}\left(e^{-2s}+(s-1)e^{-s}\right)+\underbrace{\frac{-1}{s}e^{-4s}}_{\text{If } Re\{s\}>0}$$

collecting terms in e^{-rs} gives.

$$X(s)=\frac{1}{s^2}\left(2-4e^{-s}+2e^{-2s}-se^{-4s}\right) \quad \text{ROC: } Re\{s\}>0$$

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b) $x(t)=2r(t)-4r(t-1)+2r(t-2)-u(t-4)$

using table and shifting property

c) $X(s)=\frac{2}{s^2}-\frac{4}{s^2}e^{-s}+\frac{2}{s^2}e^{-2s}-\frac{1}{s}e^{-4s}$

we can write this as in part a) by collecting terms

$$X(s)=\frac{1}{s^2}\left(2-4e^{-s}+2e^{-2s}-se^{-4s}\right)$$

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2 a) $x_1(t) = tu(t) - tu(t-2) + 2u(t-2)$

$$= tu(t) - (t-2)u(t-2)$$

$$X_1(s) = \frac{1}{s^2} - \frac{1}{s^2}e^{-2s} = \frac{1}{s^2}\left(1 - e^{-2s}\right)$$

from table. ROC: $Re\{s\} > 0$.

b) $x_2(t) = x_1(10t)$

Scaling Property $X_2(s) = \frac{1}{10} X_1\left(\frac{s}{10}\right)$

$$= \frac{10}{s^2}\left(1 - e^{-\frac{1}{5}s}\right) \quad Re\{s\}>0$$

c) $x(t) = x_1'(t) + \int_{0^-}^{t} x_2(\tau)\,d\tau$

Derivative and integration properties, causal signals

$$X(s) = s X_1(s) + \frac{1}{s} X_2(s)$$

$$= \frac{1}{s}\left(1 - e^{-2s}\right) + \frac{10}{s^3}\left(1 - e^{-\frac{1}{5}s}\right)$$

d) $x(t) = t e^{-t} \cos(2t) u(t)$

Let $z(t) = e^{-t}\cos(2t) u(t)$

Then $Z(s) = \frac{s+1}{(s+1)^2 + 4}$ using Table.

$$= \frac{s+1}{s^2+2s+5}$$

Frequency Derivative property.

$$X(s) = -\frac{d}{ds} Z(s) = -\frac{(s^2+2s+5)(1) - (s+1)(2s+2)}{(s^2+2s+5)^2}$$

$$= -\frac{s^2+2s+5 - 2s^2 - 2s - 2s - 2}{(s^2+2s+5)^2}$$

$$= \frac{s^2+2s-3}{(s^2+2s+5)^2} \quad \text{ROC: } Re\{s\} > -1$$

3 a) $x(t) = 2\delta(t) + te^{-t}u(t) + 4e^{-2t}u(t)$

$X(s) = 2 + \frac{1}{(s+1)^2} + \frac{4}{s+2}$ Table Rows 1, 5, 6

$= \frac{2(s+1)^2(s+2) + (s+2) + 4(s+1)^2}{(s+1)^2(s+2)}$

$= \frac{2s^3 + 12s^2 + 19s + 10}{s^3 + 4s^2 + 5s + 2}$

b) $x(t) = (1 - e^{-4t})u(t) = u(t) - e^{-4t}u(t)$

$X(s) = \frac{1}{s} - \frac{1}{s+4}$ Table Rows 2, 5

$= \frac{s+4-s}{s(s+4)} = \frac{4}{s^2+4s}$

c) $x(t) = e^{-3t}\cos(2\pi t)u(t) + e^{-3t}\sin(2\pi t)u(t)$

$X(s) = \frac{s+3}{(s+3)^2+4\pi^2} + \frac{2\pi}{(s+3)^2+4\pi^2}$ Rows 9a, 9b

$= \frac{s+(3+2\pi)}{s^2+6s+(9+4\pi^2)}$

d) $x(t) = 10e^{-5t}\cos(\pi t + \pi/4)u(t)$

Row 10a $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$X(s) = \frac{5\sqrt{2}(s+5-\pi)}{s^2+10s+(25-\pi^2)}$

e) $x(t) = (1 + 2t + 5t^2)e^{-10t}u(t)$

$X(s) = \frac{1}{s+10} + \frac{2}{(s+10)^2} + \frac{10}{(s+10)^3}$ Row 7 and Freq derivative

$= \frac{s^2+22s+130}{s^3+30s^2+300s+1000}$

4 a) $X(s) = \dfrac{1}{s^2+5s+4} = \dfrac{A}{s+1} + \dfrac{B}{s+4}$

$A = \left.\dfrac{1}{s+4}\right|_{s=-1} = \dfrac{1}{3}$

$B = \left.\dfrac{1}{s+1}\right|_{s=-4} = -1/3$

Row 5 $x(t) = \dfrac{1}{3}\left(e^{-t} - e^{-4t}\right)u(t)$

b) $X(s) = \dfrac{1}{s^3+9s^2+24s+16} = \dfrac{1}{(s+1)(s+4)^2}$

$= \dfrac{A}{s+1} + \dfrac{B}{s+4} + \dfrac{C}{(s+4)^2}$

$A = \left.\dfrac{1}{(s+4)^2}\right|_{s=-1} = \dfrac{1}{9}$

$B = \left.\dfrac{1}{s+1}\right|_{s=-4} = -1/3$

$C = \left.\dfrac{d}{ds}\dfrac{1}{s+1}\right|_{s=-4} = -1/9$

From table

$x(t) = \dfrac{1}{3}\left(\dfrac{1}{3}e^{-t} - \left(t + \dfrac{1}{3}\right)e^{-4t}\right)u(t)$

c) $X(s) = \dfrac{5}{s^3+s^2+25s+25} = \dfrac{5}{(s+1)(s^2+25)}$

$= \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+25}$

$= \dfrac{As^2 + 25A + (Bs+C)(s+1)}{s^3+s^2+25s+25}$

$\left.\begin{array}{l} A+B=0 \\ B+C=0 \\ 25A+C=5 \end{array}\right\}$ $A = 5/26$, $B = -5/26$, $C = 5/26$

$X(s) = \dfrac{5/26}{s+1} - \dfrac{5/26\,(s-1)}{s^2+25}$ seperate second term

$= \dfrac{5}{26}\cdot\dfrac{1}{s+1} - \dfrac{5}{26}\dfrac{s}{s^2+25} + \dfrac{1}{26}\dfrac{5}{s^2+25}$

Using Table

$x(t) = \dfrac{5}{26}\left(e^{-t} - \cos(5t) + \dfrac{1}{5}\sin(5t)\right)u(t)$

4 d) $X(s) = \dfrac{s}{s^3+s^2+9s+9} = \dfrac{s}{(s+1)(s^2+9)}$

$= \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+9}$

Clear fractions and Equate coefficients.

$(A+B)s^2 + (B+C)s + 9A+C = s$

$\left.\begin{array}{l} A+B=0 \\ B+C=1 \\ 9A+C=0 \end{array}\right\} \quad A=\frac{2}{5} \quad B=-\frac{2}{5} \quad C=\frac{7}{5}$

$X(s) = \frac{2}{5}\,\frac{1}{s+1} - \underbrace{\frac{2}{5}\,\frac{s}{s^2+9} + \frac{7}{5}\cdot\frac{1}{3}\cdot\frac{3}{s^2+9}}_{\text{split second term}}$

From Table:

$x(t) = \frac{2}{5}\left(e^{-t} - \cos(3t) + \frac{7}{6}\sin(3t)\right)u(t)$

e) $X(s) = \dfrac{s+3}{s^2+6s+45} = \dfrac{s+3}{(s+3)^2+36}$ complete square in denominator.

From Table: Row 9a $\quad x(t) = e^{-3t}\cos(6t)u(t)$

f) $X(s) = \dfrac{6}{s^2+6s+45} = \dfrac{6}{(s+3)^2+36}$ same as e)

From Table Row 9b $\quad x(t) = e^{-3t}\sin(6t)u(t)$

g) $X(s) = \dfrac{s+6}{s^2+2s+6} = \dfrac{s+6}{(s+1)^2+25}$ complete square in denominator.

$= \dfrac{s+1}{(s+1)^2+25} + \dfrac{5}{(s+1)^2+25}$ split numerator.

From Table rows 9a & 9b

$x(t) = e^{-t}\left(\cos(5t) + \sin(5t)\right)u(t)$

5 $y'' + 9y' + 20y = x$ $\quad Q(D) = D^2 + 9D + 20$

$y'(0^-) = 0 \quad y(0^-) = 2$ $\quad P(D) = 1$

a) $\left[s^2 Y_0(s) - s\overset{2}{\cancel{y(0^-)}} - \overset{0}{\cancel{y'(0^-)}}\right] + \left[9s Y_0(s) - 9\overset{2}{\cancel{y(0^-)}}\right] + 20 Y_0(s) = 0$

$(s^2 + 9s + 20)\, Y_0(s) = 2s + 18$

$$Y_0(s) = \frac{2s + 18}{s^2 + 9s + 20}$$

b) $$H(s) = \frac{P(s)}{Q(s)} = \frac{1}{s^2 + 9s + 20}$$

c) $x(t) = e^{-t} u(t)$ From Table $X(s) = \frac{1}{s+1}$

$$Y(s) = H(s) X(s) = \frac{1}{(s+1)(s^2 + 9s + 20)}$$

---

6 $y'' + 2y' + 10y = x' + 3x$ $\quad y(0^-) = y'(0^-) = 0$ (zero-state)

a) Take Laplace transform (zero-state)

$(s^2 + 2s + 10)\, Y(s) = (s + 3)\, X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s + 3}{s^2 + 2s + 10}$$

6 b)

$$x(t) = (e^{-2t} + 2e^{-5t})\,u(t)$$

$$X(s) = \frac{1}{s+2} + \frac{2}{s+5} \quad \text{From Table.}$$

$$Y(s) = H(s)X(s) = \frac{s+3}{(s+2)(s^2+2s+10)} + \frac{2(s+3)}{(s+5)(s^2+2s+10)}$$

$$Y(s) = \underbrace{\frac{A}{s+2} + \frac{Bs+C}{s^2+2s+10}} + \underbrace{\frac{D}{s+5} + \frac{Es+F}{s^2+2s+10}}$$

$$\underbrace{\begin{aligned} A+B&=0 \\ 2(A+B)+C&=1 \\ 10A+2C&=3 \end{aligned}} \qquad \underbrace{\begin{aligned} D+E&=0 \\ 2D+F+5E&=2 \\ 10D+5F&=6 \end{aligned}}$$

$$A = \frac{1}{10},\ B = \frac{-1}{10},\ C = 1 \qquad D = \frac{-4}{25},\ E = \frac{4}{25},\ F = \frac{38}{25}$$

Using Rows 9a, 9b, and 5

$$y(t) = \left[\frac{1}{10}e^{-2t} - \frac{4}{25}e^{-5t}\right]u(t) +$$

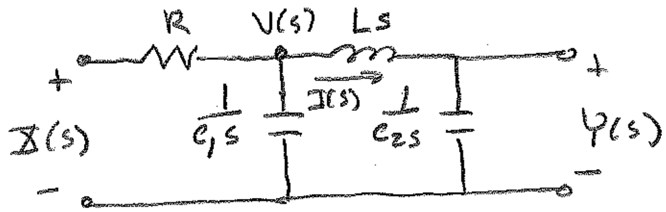
$$e^{-t}\left(-\frac{1}{10}\cos(3t) + \frac{11}{20}\sin(3t)\right)u(t) +$$

$$e^{-t}\left(\frac{4}{25}\cos(3t) + \frac{34}{25}\sin(3t)\right)u(t)$$

- Combing the cos & sin terms gives:

$$y(t) = \left[\frac{1}{10}e^{-2t} - \frac{4}{25}e^{-5t}\right]u(t) + e^{-t}\left(\frac{3}{50}\cos(3t) + \frac{301}{300}\sin(3t)\right)u(t)$$

(7) a)



b) KCL @ V(s): $\frac{X - V}{R} = C_1 s V + I$ (1)

KCL @ Y(s): $I = C_2 s Y$ (2)

KVL: $V = Ls I + Y = L C_2 s^2 Y + Y$ (3)

$X - V = R C_1 s V + R I$ rearrange (1)

$X = (1 + R C_1 s) V + R I$

substitute (2)(3)

$X = (1 + R C_1 s)(L C_2 s^2 + 1) Y + R C_2 s Y$

solve for $\frac{Y(s)}{X(s)} = H(s)$

$$\boxed{H(s) = \frac{1}{R L C_1 C_2 s^3 + L C_2 s^2 + R(C_1 + C_2) s + 1}}$$

c) $$H(s) = \frac{\frac{1}{R L C_1 C_2}}{s^3 + \frac{1}{R C_1} s^2 + \frac{C_1 + C_2}{L C_1 C_2} s + \frac{1}{R L C_1 C_2}}$$

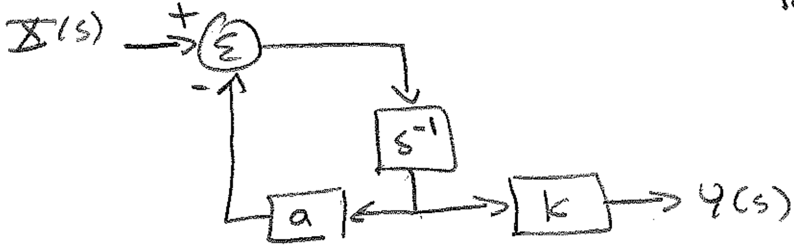
From PS #1 part b

$$y''' + \frac{1}{R C_1} y'' + \frac{\left(1 + \frac{C_2}{C_1}\right)}{L C_2} y' + \frac{1}{R C_1 C_2 L} y = \frac{1}{R C_1 C_2 L} x$$

Taking Laplace transform $H(s) = \frac{Y(s)}{X(s)}$ is same as part b)

8) a) $H(s) = \dfrac{1}{1+\frac{a}{s}} \cdot \dfrac{K}{s} = H_1(s)\,H_2(s)$

Implemented in series using feedback motif for H_1.

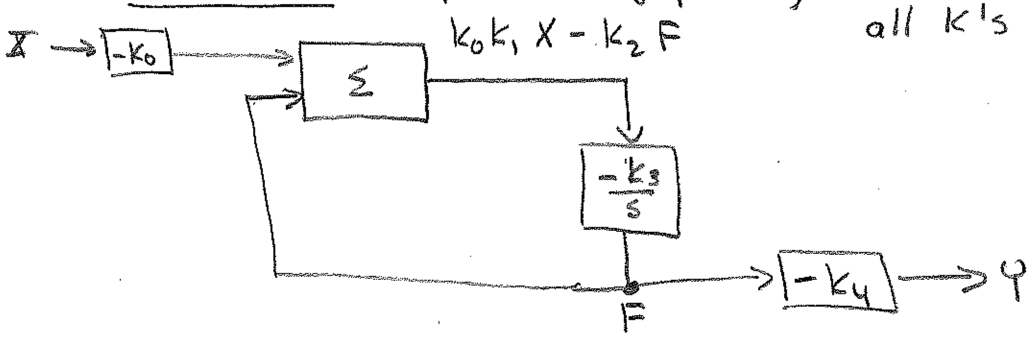


b) The general procedure is to replace each block with it's op-amp equivalent.

In the case of the feedback you need to invert either X or a

Solution 1 implementing part a)

all K's positive.



Rewriting the TF

$$F = \frac{-k_3}{s}\left(k_0 k_1 X - k_2 F\right)$$

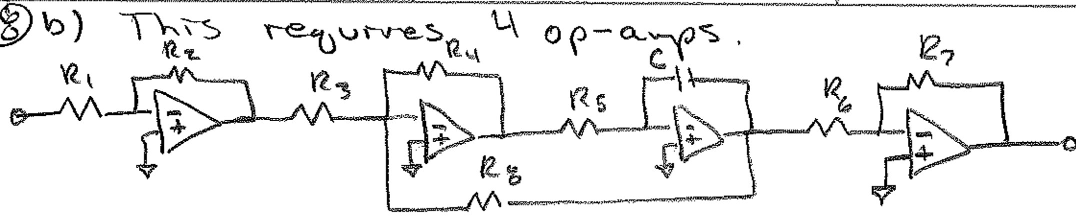
$$\frac{F}{X} = \frac{-k_0 k_1 k_3}{s + k_2 k_3}$$

$$\frac{Y}{X} = \frac{Y}{F} \cdot \frac{F}{X} = -k_4\left(\frac{-k_0 k_1 k_3}{s + k_2 k_3}\right)$$

$$= \frac{k_0 k_1 k_3 k_4}{s + k_2 k_3}$$

Let $k_0 k_1 k_3 k_4 \equiv K$ and $k_2 k_3 = a$

(8) b) This requires 4 op-amps.



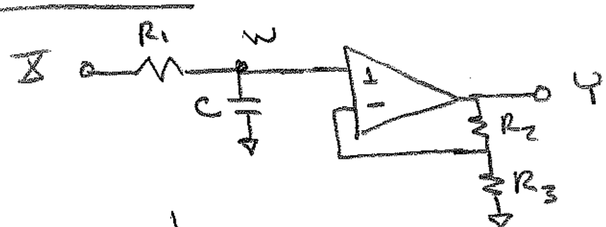
where $k_0 = \frac{R_2}{R_1}$ $k_1 = \frac{R_4}{R_3}$ $k_2 = \frac{R_4}{R_8}$ $k_3 = \frac{1}{R_5 C}$

$k_4 = \frac{R_7}{R_6}$

Note: The above is what 8 b) asked for and is a general approach to realizing TF's.

There are however specializations possible for 1^{st} order Filters that reduce the number of op-amps.

Solution 2



$$\frac{W}{X} = \frac{\frac{1}{R_1 C}}{s + \frac{1}{R_1 C}} \qquad \frac{Y}{X} = \frac{R_2 + R_3}{R_2}$$

$$H(s) = \frac{\frac{1}{R_2} \cdot \frac{1}{R_1 C} (R_2 + R_3)}{s + \frac{1}{R_1 C}}$$

Let $a = \frac{1}{R_1 C}$ then $k = \frac{1}{R_2} a (R_2 + R_3)$

This does not follow the realization procedure however.