This problem set covers the introduction to signal and systems, and time-domain methods – Chapters 1-2 of the text and Lectures 1-13.

Notes

- Problems with this icon $\boxed{\text{C}}$ can be solved with the help of a computer.
- Problems with this icon $\boxed{\text{L}}$ require the use of lab-in-a-box.

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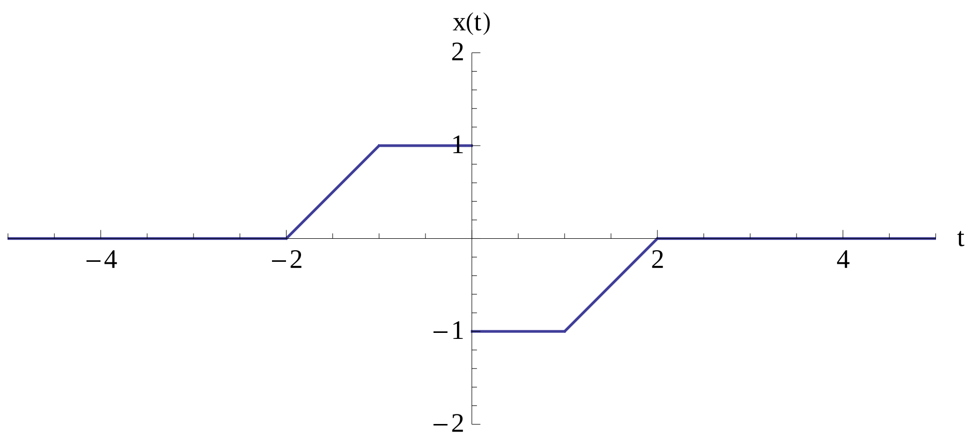
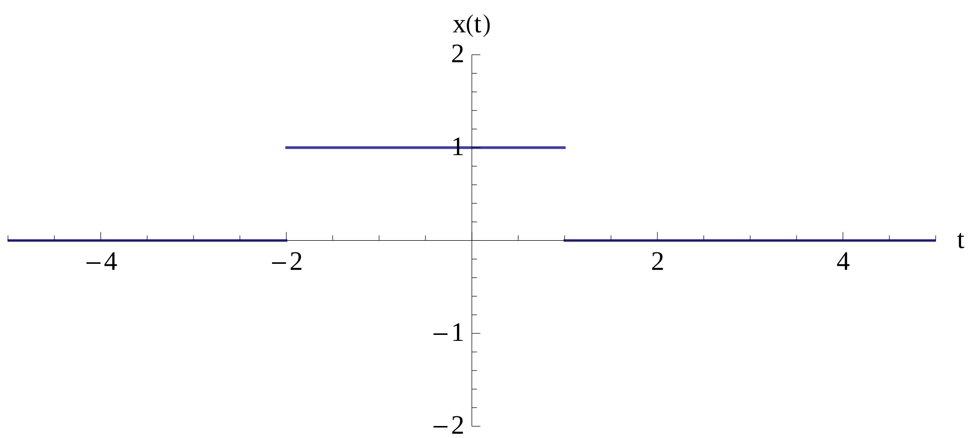
1. Sketch plots of the following signals $x(t)$ where $r(t) = t\,u(t)$ is the ramp function
   (a) (2 points) $x(t) = r(t+5) - r(t+4) - r(t-4) + r(t-5)$
   (b) (2 points) $x(t) = u(t+2) + u(t+1) - 2u(t-3) + 2e^{-(t-3)}u(t-3)$
2. Write the signals shown in terms of the impulse, step, and/or ramp, then compute their energy.



   (a) (2 points)



   (b) (2 points)

3. (2 points) Let $x(t)$ be defined as in the previous problem part b. Consider the periodic extended version of that signal

$$y(t) = \sum_{n=-\infty}^{\infty} x(t - 5n)$$

Compute the power of the signal $y(t)$.

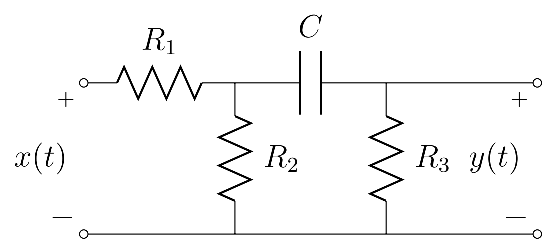
4. (2 points) $\boxed{\text{C}}$ Is the following signal an energy and/or/nor a power signal? Justify your answer.

$$x(t) = \frac{1}{2 + \cos(2\pi t)}$$

5. Which of the following signals are periodic, and if so with what fundamental period? If they are periodic compute their power.
   (a) (2 points) $x(t) = 2\cos(10t + 1)$
   (b) (2 points) $x(t) = \cos(3\pi t) + 2j\sin(6t)$
   (c) (2 points) $x(t) = 2\cos(\pi t + 2) + \sin(3\pi t)$

6. Decompose the following signals into their even and odd components.
   (a) (2 points) $u(-t) - 2u(t)$
   (b) (2 points) $\cos(2\pi t + \pi/4)$
   (c) (2 points) $\frac{1}{1+t^2}$

7. Given the following circuit,



   (a) (2 points) derive two differential equations describing the circuit, the first relating the capacitor voltage to the input, the second relating the output $y(t)$ to the capacitor voltage
   (b) (2 points) what is the capacitor voltage and the output (as piecewise functions) if the signal from problem 2 b) was applied as the input and $R_1 = 3$, $R_2 = 6$, $R_3 = 1$, and $C = \frac{2}{3}$?
   (c) (2 points) $\boxed{\text{C}}$ Plot the input, capacitor voltage, and the output (as piecewise functions)
   (d) (2 points) write the solution to part b) using the step function

8. For each of the following systems state if they are linear/nonlinear, time invariant/time varying, dynamic or instantaneous (x, y, and z are functions of time, t).
   (a) (2 points) $5y'' + 2y' + 9y = x$
   (b) (2 points) $y = t^2 x$
   (c) (2 points) $y' + y^2 = x$
   (d) (2 points) $y'' + y' - ty = x' - 2x$

9. Given a system described by the following differential equation

$$y'' + 9y' + 20y = x$$

   where $y'(0^-) = 0$ and $y(0^-) = 2$.
   (a) (2 points) find the zero-input response
   (b) (2 points) determine the impulse response
   (c) (2 points) determine the zero-state response for $x(t) = te^{-t}u(t)$.
   (d) (2 points) determine the BIBO stability of the system
   (e) (2 points) determine the asymptotic (Lyapunov) stability of the system

10. Given a system described by the following differential equation

$$y'' + 2y' + 10y = x' + 3x$$

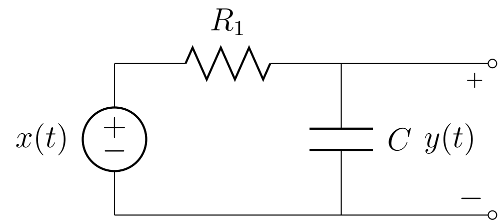
   where $y'(0^-) = 1$ and $y(0^-) = -1$.
   (a) (2 points) find the zero-input response
   (b) (2 points) determine the impulse response
   (c) (2 points) determine the zero-state response for $x(t) = e^{-2t}u(t)$.
   (d) (2 points) determine the BIBO stability of the system
   (e) (2 points) determine the asymptotic (Lyapunov) stability of the system

11. Given a system described by the following differential equation

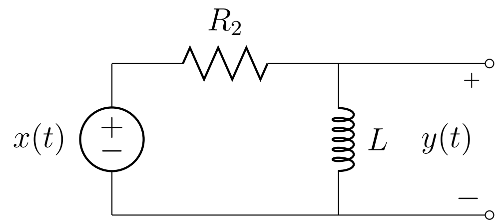
$$y' + 2y = 3x' + 6x$$

   (a) (2 points) determine the impulse response
   (b) (2 points) determine the zero-state response for $x(t) = u(t)$.
   (c) (2 points) determine the BIBO stability of the system
   (d) (2 points) determine the asymptotic (Lyapunov) stability of the system

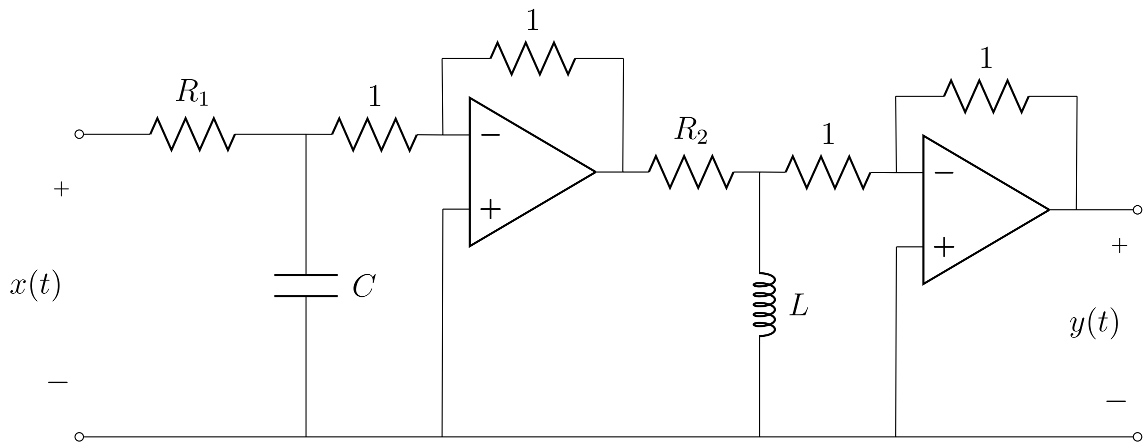
12. Given the following circuits, where $R_1 = 2$, $C = \frac{1}{2}$, $R_2 = 4$, and $L = 2$



Circuit 1



Circuit 2



Circuit 3

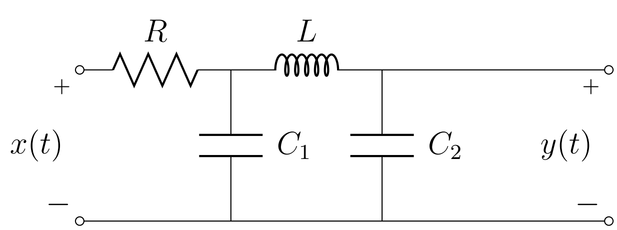
(a) (2 points) determine the impulse response of circuit 1

(b) (2 points) determine the impulse response of circuit 2

(c) (2 points) determine the impulse response of circuit 3

(d) (2 points) show that the impulse response of circuit 3 is the convolution of the individual impulse responses from circuits 1 and 2.

13. Given the following circuit,



(a) (2 points) determine the state equations for the system

(b) (2 points) determine the overall differential equation describing the input-output relationship in terms of R, C_1, C_2 and L

(c) (2 points) $\boxed{\text{C}}$ determine its impulse response using $R = 10\ \Omega$, $C_1 = 1\ \mu$F, $C_2 = 2.2\ \mu$F, and $L = 1$mH.

14. $\boxed{\text{L}}$ $\boxed{\text{C}}$ Construct the circuit from Problem 13 using $R = 10\ \Omega$, $C_1 = 1\ \mu\text{F}$, $C_2 = 2.2\ \mu\text{F}$, and $L = 1\text{mH}$.

  (a) (2 points) using your result from Problem 13.c, determine the expected voltage across C_2 when $x(t) = u(t)$.

  (b) (2 points) again, using your result from Problem 13.c, estimate the system time constant, τ.

  (c) (2 points) apply a (0-1)V square wave to the input of the circuit with a period of approximately 4τ and 50% duty cycle. Measure the output voltage waveform over $\frac{1}{2}$ the cycle (the non-zero half).

  (d) (2 points) plot the recorded input and output waveforms from part c)

  (e) (2 points) explain your results in d) in terms of parts a) and b).

  Note, inductors are the most non-ideal circuit elements in practice. Your analysis in part e) might need to take that into account.