The purpose of this first problem set is to remind you of the material you need from prerequisite courses. You should be able to do these problems with minimal difficulty in 4-6 hours (perhaps after refreshing your memory from old notes or a text). If you have trouble with these problems you should do more examples or seek help from the TA or myself as soon as possible. This course uses these concepts from the first few lectures through the end of the semester, so it is important that you understand this material.

Notes

- Problems with this icon $\boxed{\text{C}}$ should be solved with the help of a computer.
- Problems with this icon $\boxed{\text{L}}$ require the use of lab-in-a-box.

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1. Integration: material from Math 1205/1206.
   (a) (2 points) Breaking a definite integral into regions:

$$f(x) = \begin{cases} e^{x} & x < 0 \\ e^{-x} & x \geq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\, dx =$$

   (b) (2 points) $\boxed{\text{C}}$ Integration by substitution using integration tables (or a computer algebra system):

$$\int \frac{1}{a^2 + x^2}\, dx =$$

   (c) (2 points) Integration by parts:

$$\int x e^{x}\, dx =$$

   (d) (2 points) Integration involving trig functions:

$$\int \sin x\, dx =$$

   (e) (2 points) Integration involving trig functions over definite limits:

$$\int_{0}^{2\pi} \sin(x)\cos(ax)\, dx =$$

2. Complex Numbers and Phasors: material from ECE 2004. Note we use the engineering notation $j = \sqrt{-1}$. $|.|$ denotes magnitude, $\angle$ denotes phase or angle, and $*$ denotes conjugate.

   (a) (2 points) Complex number representation:

   $$\text{Sketch a diagram of } 3 + j4$$

   (b) (2 points) Operations: $c = a + jb$, $d = f + jg$

   $$\begin{aligned} \Re(c) &= \\ \Im(c) &= \\ (c)^* &= \\ \left|\frac{c}{d}\right| &= \\ \angle\left(\frac{c}{d}\right) &= \end{aligned}$$

   (c) (2 points) Phasors: Write the following as two oppositely rotating phasors using Euler's identity.

   $$120\sin\left(2\pi t - \frac{\pi}{6}\right) \quad -\infty < t < \infty$$
   $$\cos\left(\frac{9}{2}\pi t - \frac{2}{3}\pi\right) \quad -\infty < t < \infty$$

3. Differential Equations: material from Math 2214. We will use the common engineering notation

   $$\dot{x} = x' = \frac{dx}{dt} \text{ and } \ddot{x} = x'' = \frac{d^2x}{dt^2}$$

   (a) (2 points) Linear Differential Equations of 1st order: Solve the following for $x(t)$:

   $$\dot{x} - 3x = e^{3t} \text{ with } x(0) = 4$$

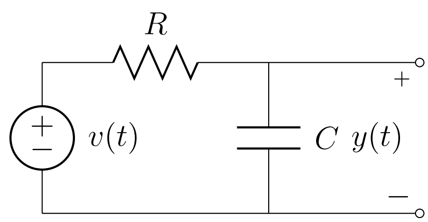
   (b) (2 points) Linear Differential Equations of 2nd order: Solve the following for $x(t)$:

   $$\ddot{x} + 6\dot{x} + 8x = 0 \text{ with } x(0) = 2\ ,\ \dot{x}(0) = 0$$

4. Circuits: material from ECE 2004. Note, passive element values are chosen to make the arithmetic simple and are not realistic. The constant V refers to a generic DC voltage.

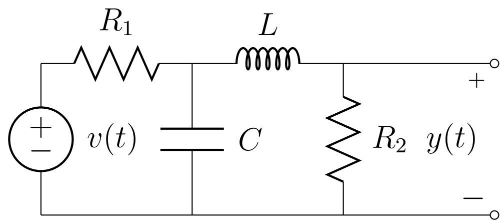
   (a) (2 points) Consider the following circuit where $R = 2$ and $C = \frac{1}{2}$. Suppose the input voltage is given by

   $$v(t) = \begin{cases} V & t \leq 0^- \\ V\cos(2\pi t) & t \geq 0^+ \end{cases}$$



Find the voltage $y(t)$ for all t.

(b) (2 points) Consider the following second order circuit where $R_1 = 2$, $R_2 = 3$, $C = \frac{1}{2}$, and $L = 2$.

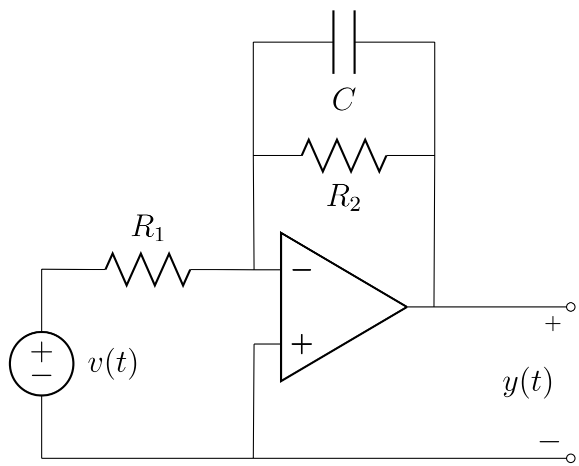


where

$$v(t) = \begin{cases} V & t \leq 0^- \\ 0 & t \geq 0^+ \end{cases}$$

Find the voltage $y(t)$ for all t.

(c) (2 points) In the following circuit, assume an ideal operational amplifier and find $y(t)$ as a function of R_1, R_2, C, and $v(t)$.



(d) (2 points) $\boxed{\text{C}}$ For the circuit in problem 4.c above find values of the components such that the output is the (negative) integral of the input. Plot the input and output on the same plot when

$$v(t) = \begin{cases} 0 & t \leq 0^- \\ Ve^{-t} & t \geq 0^+ \end{cases}$$

(e) (2 points) $\boxed{\text{L}}$ Build the circuit from problem 4.a using values of R and C such that $RC \approx 1$ milli-Ohm-Farad (0.001). Apply a square wave input of amplitude $0 - 5$ volts, period of 10ms, and a 50% duty cycle as the input $x(t)$ and measure the output voltage $y(t)$ using your function generator and oscilloscope for one period.

To plot the experimental result in Matlab, you can export the data from the oscilloscope software in csv format, then import the file into Matlab and plot the results. Click Export in the toolbar, save the file in .csv format. Import the data into Matlab by clicking on import data in the toolbar, select the file you have saved in .csv format and click on Import Selection. The data will be available in your workspace for plotting.