1.a) $f(x) = \begin{cases} e^{x} & x < 0 \\ e^{-x} & x \geq 0 \end{cases}$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0^-} e^{x}\,dx + \int_{0}^{\infty} e^{-x}\,dx$$

Break integral into regions $(-\infty, 0)$ and $(0, \infty)$

$$= e^{x}\Big|_{-\infty}^{0^-} + -e^{-x}\Big|_{0}^{\infty} = (1-0) + (-0+1) = \boxed{2}$$

1.b) $$\int \frac{1}{a^2+x^2}\,dx = \boxed{\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C}$$

From table or CAS.

1.c) $\int x e^{x}\,dx$ Let $v = e^{x}$ $u = x$
$dv = e^{x}$ $du = 1$

Integration by parts.

$$\int uv = uv' - \int u'v$$

$$\int x e^{x}\,dx = x e^{x} - \int e^{x}\,dx$$

$$\boxed{= e^{x}(x-1) + C}$$

1.d) $$\int \sin x\,dx = \boxed{-\cos(x) + C}$$

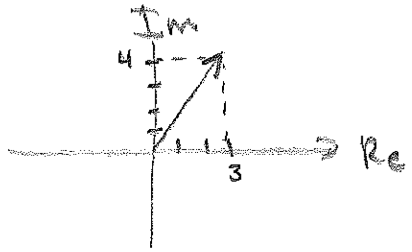
Table / trivial
$\frac{d}{dt}\cos = -\sin$

1.e) $$\int_0^{2\pi} \sin(x)\cos(ax)\,dx = \frac{\cos(x)\cos(ax) + a\sin(x)\sin(ax)}{a^2-1}\Bigg|_0^{2\pi}$$

Table or using trig identity for sin cos product

$$\boxed{= \frac{\cos(2a\pi)}{a^2-1} - \frac{1}{a^2-1} = \frac{\cos(2a\pi)-1}{a^2-1} = \frac{-2\sin^2(a\pi)}{a^2-1}}$$

2.a)



2.b) $c = a + jb \quad d = f + jg$

$\mathrm{Re}(c) = a$

$\mathrm{Im}(c) = b$

$c^* = a - jb$

$$\left|\frac{c}{d}\right| = \left|\frac{a+jb}{f+jg}\right| = \frac{(a^2+b^2)^{1/2}}{(f^2+g^2)^{1/2}}$$

$$\angle \frac{c}{d} = \angle c - \angle d = \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(\frac{g}{f}\right)$$

2.c)

$$120\sin(2\pi t - \pi/6) = 120\cos(2\pi t - \frac{\pi}{6} + \pi/2)$$

$$= 120\cos(2\pi t + \pi/3)$$

$\cos\theta = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$

$$\boxed{= 60e^{j2\pi t + \pi/3} + 60e^{-j2\pi t - \pi/3}}$$

$$= 60\angle \pi/3 + 60\angle -\pi/3$$

rotating ccw   rotating cw

$$\cos\left(\frac{9}{2}\pi t - \frac{2\pi}{3}\right) = \frac{1}{2}e^{j\frac{9}{2}\pi t - \frac{2}{3}\pi} + \frac{1}{2}e^{-j\frac{9}{2}\pi t + \frac{2\pi}{3}}$$

$$= \frac{1}{2}\angle -\tfrac{2}{3}\pi + \frac{1}{2}\angle \frac{2\pi}{3}$$

rotating ccw   rotating cw

3,a) $\dot{x} - 3x = e^{3t} \qquad x(0) = 4$

multiply through by e^{-3t}

$$\underbrace{e^{-3t}\dot{x} - 3e^{-3t}x}_{} = e^{3t}e^{-3t} = 1$$

$$\frac{d}{dt}\left[xe^{-3t}\right] = 1 \qquad \int \text{both sides}$$

$$xe^{-3t} = \int 1\,dt = t + C$$

multiply trough by e^{3t}

$$x = te^{3t} + Ce^{3t}$$

$$x(0) = C = 4$$

$$\boxed{x(t) = te^{3t} + 4e^{3t}}$$

3.b) $\ddot{x} + 6\dot{x} + 8x = 0 \qquad x(0) = 2 \qquad \dot{x}(0) = 0$

Characteristic equation is $\lambda^2 + 6\lambda + 8 = 0$
with roots $\lambda = -4$ AND $\lambda = -2$

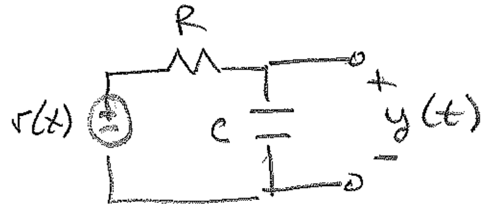
$$x(t) = C_1e^{-4t} + C_2e^{-2t}$$

$$\left.\begin{aligned} x(0) &= C_1 + C_2 = 2 \\ \dot{x}(0) &= -4C_1 - 2C_2 = 0 \end{aligned}\right\}$$

solve for C_1, C_2
$C_1 = -2 \quad C_2 = 4$

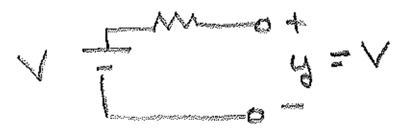
$$\boxed{x(t) = -2e^{-4t} + 4e^{-2t}}$$

4.a)



$$v(t) = \begin{cases} V & t < 0 \\ V\cos(2\pi t) & t \geq 0^+ \end{cases}$$

For $t < 0$



capacitor is open
no current flows

For $t > 0$

$$\frac{v(t) - y(t)}{R} = Cy'(t) \qquad \text{KCL}$$

$$y' + \frac{1}{RC} y = \frac{V}{RC}\cos(2\pi t)$$ rearranging and substitute v.

with $y(0) = V$

Let $\alpha = \frac{1}{RC} \Rightarrow y' + \alpha y = \alpha V\cos(2\pi t) \quad \alpha > 0$

$y(t)$ = Homogeneous solution. + particular solution

Homogeneous solution

$y_h' + \alpha y_h = 0 \Rightarrow y_h = Ae^{-\alpha t}$ for some constant A.

Particular Solution is of the form

$y_p = B\cos(\omega t) + C\sin(\omega t)$ with $\omega = 2\pi$

taking derivative and substituting

$$y_p' + \alpha y_p = (\underbrace{2\pi C + \alpha B}_{\alpha V})\cos(2\pi t) + (\underbrace{\alpha C - 2\pi B}_{0})\sin(2\pi t)$$
$$= \alpha V \cos(2\pi t) + 0 \sin(2\pi t)$$

$2\pi C + \alpha B = \alpha V$ AND $\alpha C - 2\pi B = 0 \Rightarrow B = \frac{\alpha^2 V}{4\pi^2 + \alpha^2}$

$$C = \frac{2\pi\alpha V}{4\pi^2 + \alpha^2}$$

AND $y(t) = y_h(t) + y_p(t)$

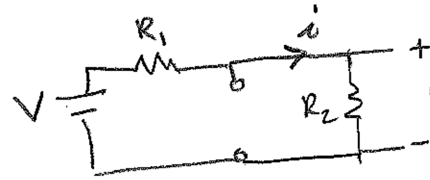
4.a cont.) To solve for the constant A in y_h

use $y(0) = A + \frac{\alpha^2 V}{4\pi^2+\alpha^2} = V \Rightarrow A = \frac{4\pi^2 V}{4\pi^2+\alpha^2}$

Thus the final Solution is

$$\boxed{y(t) = \begin{cases} V & t<0 \\ \frac{4\pi^2 V}{4\pi^2+\alpha^2} e^{-\alpha t} + \frac{\alpha^2 V}{4\pi^2+\alpha^2}\cos(2\pi t) + \frac{2\pi\alpha V}{4\pi^2+\alpha^2}\sin(2\pi t) & t \geq 0 \end{cases}}$$

4.b) For $t<0$



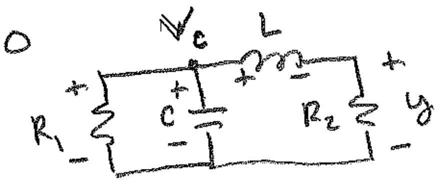
$y = \frac{R_2}{R_1+R_2} V$ voltage divider

$= \frac{3}{5} V$

so $\underline{y(0) = \frac{3}{5} V}$, $y = R_2 i \Rightarrow y' = R_2 i' \Rightarrow \underline{y'(0) = 0}$

current cannot change instantaneously

For $t>0$



① KCL at Vc $\quad \frac{V_c}{R_1} + C V_c' = i$

② KVL $\quad L i' + y = V_c$

③ Ohms Law $\quad y = R_2 i$

③ → ② gives $\quad V_c = \frac{L}{R_2} y' + y$ ④

③ + ④ → ① gives

$$y'' + \frac{R_1 R_2 C + L}{R_1 L C} + \frac{R_1+R_2}{R_1 L C} = 0$$

$R_1 = 2,\ R_2 = 3,\ L = 2,\ C = 1/2$

$$y'' + \frac{5}{2} y' + \frac{5}{2} y = 0$$

4.b cont.) The characteristic Eq is $\mu^2 + \frac{5}{2}\mu + \frac{5}{2}$

with roots $\mu_{1,2} = -\frac{5}{4} \pm j\frac{\sqrt{15}}{4}$

Since the roots are complex the form of the homogeneous solution is

$$y(t) = e^{-\frac{5}{4}t}\left(A\cos\left(\frac{\sqrt{15}}{4}t\right) + B\sin\left(\frac{\sqrt{15}}{4}t\right)\right)$$

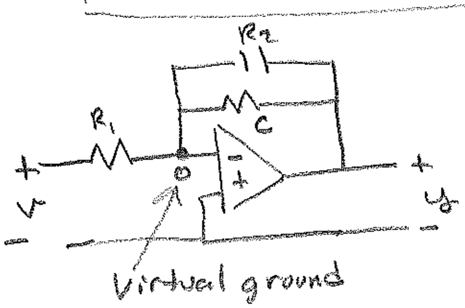
$$y(0) = A = \frac{3}{5}\,V$$

$$y'(0) = \frac{\sqrt{15}}{4}B - \frac{5}{4}A = 0 \Rightarrow B = \frac{3}{\sqrt{15}}\,V$$

$\therefore$ The total solution for all time is

$$\boxed{y(t) = \begin{cases} \frac{3}{5}V & t < 0 \\ e^{-\frac{5}{4}t}\left(\frac{3}{5}V\cos\left(\frac{\sqrt{15}}{4}t\right) + \frac{3}{\sqrt{15}}V\sin\left(\frac{\sqrt{15}}{4}t\right)\right) & t > 0 \end{cases}}$$

4.c)



KCL @ inverting input

$$\frac{v}{R_1} + Cy' + \frac{y}{R_2} = 0$$

or

$$y' + \frac{1}{R_2C}y = \frac{-1}{R_1C}v$$

This is a 1st order DE so,

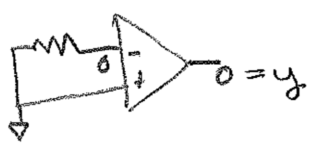
$$\boxed{y(t) = \frac{-1}{R_1C}e^{-\frac{1}{R_2C}t}\int_{-\infty}^{t} e^{\frac{1}{R_2C}\tau}v(\tau)\,d\tau}$$

4.d) From problem 4.c) if $R_1 \cdot C = 1$ and $R_2 \to \infty$ then

$$y(t) = -\int_{-\infty}^{t} v(\tau)\, d\tau$$

For $v(t) = \begin{cases} 0 & t < 0 \\ Ve^{-t} & t > 0 \end{cases}$

when $t < 0$



when $t > 0$

$$y(t) = -\int_{0}^{t} Ve^{-\tau}\, d\tau = Ve^{-t}\Big|_{0}^{t}$$

$$= Ve^{-t} - V$$
$$= V(e^{-t} - 1)$$

putting $y(t)$ together yields

$$y(t) = \begin{cases} 0 & t < 0 \\ V(e^{-t} - 1) & t > 0 \end{cases}$$

Plotting