The purpose of this first problem set is to remind you of the material you need from prerequisite courses. You should be able to do these problems with minimal difficulty in 3-4 hours (perhaps after refreshing your memory from old notes or a text). If you have trouble with these problems you should do more examples or seek help from the TA or myself at the first help session. This course uses these concepts from the first few lectures through the end of the semester, so it is important that you understand this material.

Note: In the following $a, b$ are constant real numbers.

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**Integration**

Material from Math 1205/1206.

1. Breaking a definite integral into regions:

   \[ f(x) = \begin{cases} 
   a^x & x < 0 \\
   a^{-x} & x \geq 0
   \end{cases} \quad a > 0 \]

   \[ \int_{-\infty}^{\infty} f(x) \, dx = \]

2. Integration by substitution using integration tables:

   \[ \int \frac{1}{a^2 + x^2} \, dx = \]

3. Integration by parts:

   \[ \int xe^x \, dx = \]

4. Integration involving trig functions:

   \[ \int \sin x \, dx = \]

5. Integration of rational functions:

   \[ \int \frac{3x - 1}{x^2 - x - 6} \, dx = \]

6. Integration involving the natural log and exponential:

   \[ \int_0^1 x \log x \, dx = \]
Complex Numbers

Material from ECE 2004. Note we use the engineering notation $j = \sqrt{-1}$. $|\cdot|$ denotes magnitude, $\angle$ denotes phase or angle, and $*$ denotes conjugate. Phasors are projected onto the real axis.

7. Complex number representation:

Sketch a diagram of $3 + j4$

8. Operations: $c = a + jb$

\[
\begin{align*}
\Re(c) & = \\
\Im(c) & = \\
|c| & = \\
\angle(c) & = \\
(c)^* & =
\end{align*}
\]

9. Phasors: Write the following as two oppositely rotating phasors.

\[120 \sin \left(2\pi t - \frac{\pi}{6}\right) - \infty < t < \infty\]

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Differential Equations

Material from Math 2214. We will use the common engineering notation

\[
\dot{x} = x' = \frac{dx}{dt} \quad \text{and} \quad \ddot{x} = x'' = \frac{d^2x}{dt^2}
\]

10. Linear Differential Equations of 1st order: Solve the following for $x(t)$:

\[
\dot{x} - 3x = te^{3t} \quad \text{with} \quad x(0) = 4
\]

11. Linear Differential Equations of 2nd order: Solve the following for $x(t)$:

\[
\dddot{x} - 4\dot{x} + 13x = 0 \quad \text{with} \quad x(0) = 4, \ \dot{x}(0) = 10
\]
Circuits

Material from ECE 2004. Note, passive element values are chosen to make the arithmetic simple and are not realistic.

12. Linear 1st order circuits: For the circuit below, the current through the inductor is given by

\[ i(t) = \begin{cases} 
0 & \text{for } -\infty < t < 0 \\
1 - e^{-2t} & \text{for } t > 0 
\end{cases} \]

Find \( V(t) \) for \( t > 0 \) in terms of \( R \) and \( L \).

![Circuit Diagram]

13. Linear 2nd order circuits: Given the following circuit, where both switches are opened at \( t = 0 \), find the current through the inductor and the voltage across the capacitor for \( t > 0 \).

![Circuit Diagram]

14. OpAmps: Given the following circuit, where the switch is moved at \( t = 0 \) and \( U1 \) is an ideal opamp (infinite gain), find the voltage \( V_o \) in terms of \( V, R1, R2, \) and \( C1 \).
1. \[ f(x) = \begin{cases} a^x & x < 0 \\ \frac{1}{a^{-x}} & x > 0 \end{cases} \]

\[ \int f(x) \, dx = \int_{-\infty}^{0} a^x \, dx + \int_{0}^{\infty} a^{-x} \, dx \]

\[ = \frac{a^x}{\ln a} \bigg|_{-\infty}^{0} + \frac{a^{-x}}{-\ln a} \bigg|_{0}^{\infty} \]

\[ = \frac{1}{\ln a} + 0 - 0 - \frac{1}{-\ln a} \]

\[ = \frac{1}{\ln a} + \frac{1}{\ln a} = \frac{2}{\ln a} \]

2. \[ \int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C \] from table

3. \[ \int x e^x \, dx = \text{let} \quad v = e^x \quad u = x \]

\[ dv = e^x \quad du = 1 \]

\[ = x e^x - \int e^x \, dx \quad \text{integration by parts} \]

\[ = e^x (x-1) + C \]
\[ \int \sin x \, dx = -\cos(x) + C \quad \text{Table.} \]

\[ \int \frac{3x-1}{x^2-x-6} \, dx \]

Factoring.

\[ \frac{3x-1}{x^2-x-6} = \frac{A}{x+2} + \frac{B}{x-3} \]

\[ 3x-1 = A(x-3) + B(x+2) \]

\[ \text{Solves} \]

\[ A = \frac{7}{5}, \quad B = \frac{8}{5} \]

\[ = \frac{7}{5} \ln |x+2| + \frac{8}{5} \ln |x-3| + C. \]

\[ \int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C \quad \text{From Table by parts.} \]

\[ = \left( 0 - \frac{1}{4} \right) - \left( \frac{0^2}{2} \ln 0 - 0 \right) \]

\[ \lim_{\epsilon \to 0^+} \frac{e^2}{2} \ln \epsilon = \lim_{\epsilon \to 0^-} \frac{-1}{2 \epsilon^3} = \lim_{\epsilon \to 0^+} \frac{e^2}{-2} = 0 \]

\[ = -\frac{1}{4} \]
\( \mathbf{7} \quad 3 + j 4 \)

\begin{align*}
\text{Re} \, 3 + j 4 &= 3 \\
\text{Im} \, 3 + j 4 &= 4 \\
|z| &= (3^2 + 4^2)^{1/2} \\
\angle z &= \tan^{-1} \frac{4}{3} \\
\bar{z} &= 3 - j 4
\end{align*}

\( \mathbf{8} \quad c = a + j b \\
\text{Re} \, c^2 = a \\
\text{Im} \, c^2 = b \\
|z| = (a^2 + b^2)^{1/2} \\
\angle z = \tan^{-1} \frac{b}{a} \\
\bar{z} = a - j b
\)

\( \mathbf{9} \quad 120 \sin \left( 2\pi t - \frac{\pi}{6} \right) = 120 \cos \left( 2\pi t - \frac{\pi}{6} + \frac{\pi}{2} \right) \\
= 120 \cos \left( 2\pi t + \frac{\pi}{3} \right) \\
= 60e^{j\left(2\pi t + \frac{\pi}{3}\right)} + 60e^{j\left(2\pi t + \frac{\pi}{3}\right)}
\)

\( \mathbf{10} \quad x = 3x = te^{3t} \\
\text{multiply through by} \quad e^{-3t} = e^{3t} \\
x - 3xe^{-3t} = t \\
\frac{d}{dt} \left[ e^{-3t} x \right] = t \\
e^{-3t} x = \int t \, dt = \frac{1}{2} t^2 + C \\
x = \frac{1}{2} t^2 e^{3t} + Ce^{3t}
\)

\( x(0) = C = 1 \\
x(t) = \frac{1}{2} t^2 e^{3t} + 4e^{3t} \)
(11) \[ x - 4x + 13x = 0 \]

Auxiliary equation \[ r^2 - 4r + 13 = 0 \]

\[ r = \frac{4 \pm \sqrt{16 - 52}}{2} \]

\[ = 2 \pm j3 \]

\[ x(t) = C_1 e^{zt} \cos(zt) + C_2 e^{zt} \sin(zt) \]

\[ x(0) = C_1 = 4 \]

\[ x(0) = 2C_1 + 6C_2 = 10 \quad \Rightarrow \quad C_2 = \frac{1}{3} \]

\[ x(t) = 4e^{zt} \cos(zt) + \frac{1}{3} e^{zt} \sin(zt) \]

(12) \[ \dot{i}(t) = \begin{cases} 0 & t < 0 \\ 1 - 2e^{-2t} & t > 0 \end{cases} \]

Initial condition \[ i(0) = 0 \quad v(t) = 0 \]

KVL: \[ v(t) = R\dot{i}(t) + L \frac{di}{dt} \]

\[ \frac{di}{dt} = 2e^{-2t} \quad \text{for} \ t > 0 \]

\[ v(t) = R(1 - e^{-2t}) + 2Le^{-2t} \quad \text{for} \ t > 0 \]
for $t = 0^-$

\[
V_c = \frac{10}{s + 10} \quad q = 2 \quad \dot{q} = V_c = 1
\]

\[
\begin{align*}
\text{for } t = 0^+ \\
K L & - 5 \ddot{z} = \frac{1}{2} \frac{d^2 \dot{z}}{dt^2} + V_c \quad \forall t \\
- V_c & = \frac{1}{2} \frac{d^2 \dot{z}}{dt^2} \quad + 5 \ddot{z} \\
- 8 \int_0^t \dot{z} \, dt & = \frac{1}{2} \frac{d^2 \dot{z}}{dt^2} + 5 \ddot{z} \\
\frac{d^2 \dot{z}}{dt^2} & = 10 \frac{d \dot{z}}{dt} + 16 \ddot{z} = 0.
\end{align*}
\]

Characteristic Eq. $s^2 + 10s + 16$ has roots $r = -2, -8$.

\[
\begin{align*}
\ddot{z}(t) & = c_1 e^{-2t} + c_2 e^{-8t} \\
\dot{z}(0) & = c_1 + c_2 = 1, \quad \frac{d \dot{z}}{dt} = -10 \ddot{z} - 2V_c \\
& \quad \bigg|_{t = 0} = -14 \\
& \quad -2 \dot{c}_1 - 8 \dot{c}_2 = 14.
\end{align*}
\]

Above $\Rightarrow c_1 = -1, c_2 = 2$.

\[
\ddot{z}(t) = -2e^{-2t} - 8e^{-8t} \quad t > 0
\]
(13) cont.

\[ V_e(t) = -\frac{1}{2} \frac{d^2}{dt^2} - 5x(t) \]

\[ = -\frac{1}{2} \left( -16 e^{-8t} + 2 e^{-2t} \right) - 5 \left( 2 e^{-8t} - e^{-2t} \right) \]

\[ V_0(t) = -2 e^{-8t} + 4 e^{-2t}, \quad t > 0 \]

(14)

\[ \frac{V}{R_1} = -\frac{V_0}{R_2} \rightarrow V_0 = -\frac{R_2}{R_1} V \]

\[ t = 0^+ \]

\[ C \frac{dV_c}{dt} + \frac{V_c}{R_2} = 0 \]

\[ V_c = V_0 \]

\[ \frac{dV_c}{dt} + \frac{C}{R_2} V_c = 0 \]

\[ V_c(t) = V_c(0) e^{-\frac{C}{R_2} t} \]