★ Review of Time Domain Methods.

Summary:

- time domain signals are functions of a real variable $t$ (time) that are continuous or have a finite number of discontinuities.

  Examples: $u(t)$, $\delta(t)$, $e^{\lambda t}u(t)$, $\cos(\omega t)$

  $\cos(\omega t)u(t)$, $e^{\lambda t}\cos(\omega_0 t)$, $e^{t^2}$, etc.

- Linear, Time Invariant (LTI) systems are described by linear, constant coefficient, differential Equations with zero initial conditions.

$$D^N y + a_1 D^{N-1} y + \cdots a_N y = b_0 D^N x + b_1 D^{N-1} x + \cdots b_N x$$

$$D^N y = \frac{d^N y}{dt^N} \quad \text{or} \quad Q(D)\, y = P(D)\, x.$$

  or equivalently by the impulse response $h(t)$, the solution to $Q(D)\,y = P(D)\,\delta(t)$

- the zero-state response to an input $x(t)$

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

  is the convolution of $x$ & $h$

$$y(t) = (h * x)(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

- A system is BIBO stable if $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$.
- A system is Internally stable if roots $Q(D)=0$ are all in left hand of complex-plane.
- The response to a stable system consists of:

$$y(t) = y_{zero\text{-}input}(t) + y_{transient}(t) + y_{steady\text{-}state}(t)$$

- The time it takes for the transient response to decay is the system time-constant.

Example #1

$$y'' + 7y' + 6y = x' + 2x$$

$y(0^-) = 1$, $y'(0^-) = 1$

using time-domain methods, determine:

a) zero-input response

$Q(D) = D^2 + 7D + 6$

↳ roots $\frac{-7 \pm \sqrt{49-32}}{2} = -6, -1$

$y(t) = c_1 e^{-6t} + c_2 e^{-t}$

$y'(t) = -6c_1 e^{-6t} - c_2 e^{-t}$

$$\left.\begin{aligned} y(0^-) &= c_1 + c_2 = 1 \\ y'(0^-) &= -6c_1 - c_2 = 1 \end{aligned}\right\} \quad \begin{aligned} c_2 &= 1 - c_1 = \tfrac{7}{5} \\ -6c_1 - (1 - c_1) &= 1 \\ -6c_1 + c_1 &= 2 \\ c_1 &= -\tfrac{2}{5} \end{aligned}$$

$\therefore \; y_0(t) = -\frac{2}{5} e^{-6t} u(t) + \frac{7}{5} e^{-t} u(t)$

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b) impulse response. $P(D) = D + 2 = \underset{b_0}{\underset{\Downarrow}{0}}D^2 + D + 2$

$h(t) = b_0 \delta(t) + P(D) y_n(t) u(t)$

$y_n(t) = c_1 e^{-6t} + c_2 e^{-t}$ with $y_n(0^+) = 0$, $y_n'(0^+) = 1$

$$\text{Solve: } \left.\begin{aligned} c_1 + c_2 &= 0 \\ -6c_1 - c_2 &= 1 \end{aligned}\right\} \quad \begin{aligned} c_1 &= -1/5 \\ c_2 &= 1/5 \end{aligned}$$

$y_n(t) = -\frac{1}{5} e^{-6t} + \frac{1}{5} e^{-t}$

$P(D) y_n(t) = +\frac{6}{5} e^{-6t} - \frac{1}{5} e^{-t} - \frac{2}{5} e^{-6t} + \frac{2}{5} e^{-t}$

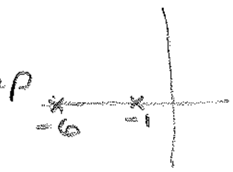
$= \frac{4}{5} e^{-6t} + \frac{1}{5} e^{-t}$

$h(t) = \left(\frac{4}{5} e^{-6t} + \frac{1}{5} e^{-t}\right) u(t)$

Example #1 cont.

e) Determine the internal stability.

roots of $Q(D) = -6, -1$ are in LHP



$\therefore$ <u>internally stable</u>

— Example #2. Given $h(t) = (e^{2t} + e^{-2t})\,u(t)$

Determine the zero-state response to $x(t) = u(t)$ <u>using convolution</u>.

$$y(t) = (h * x)(t) = \left[e^{2t}u(t) * u(t)\right] + \left[e^{-2t}u(t) * u(t)\right]$$

Using Table 2.1

$$y(t) = \frac{1 - e^{2t}}{-2}\,u(t) + \frac{1 - e^{-2t}}{2}\,u(t)$$

$$= \cancel{-\tfrac{1}{2}} + \tfrac{1}{2}e^{2t}u(t) + \cancel{\tfrac{1}{2}} - \tfrac{1}{2}e^{-2t}u(t)$$

$$= \tfrac{1}{2}e^{2t}u(t) - \tfrac{1}{2}e^{-2t}u(t)$$

$$= \tfrac{1}{2}\left(e^{2t} - e^{-2t}\right)u(t)$$

— Example #3 Determine if $h(t) = (e^{2t} + e^{-2t})\,u(t)$ represents a BIBO stable system.

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{0}^{\infty} \left|e^{2t} + e^{-2t}\right| dt$$

$$= \int_{0}^{\infty} e^{2t} + e^{-2t}\,dt = \int_{0}^{\infty} e^{2t}\,dt + \int_{0}^{\infty} e^{-2t}\,dt$$

$$= \tfrac{1}{2}e^{2t}\Big|_0^{\infty} + -\tfrac{1}{2}e^{-2t}\Big|_0^{\infty}$$

$$= \left(\infty - \tfrac{1}{2}\right) + \left(0 + \tfrac{1}{2}\right)$$

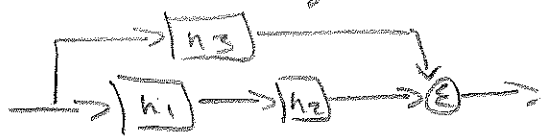
$$= \infty \quad \therefore \ \underline{\text{NOT}}\ \text{BIBO stable.}$$

(4) given $h_1(t) = e^{-3t}u(t)$

$h_2(t) = e^{-5t}u(t)$

$h_3(t) = e^{-9t}\cos(3\pi t)u(t)$

and a block diagram




What is the overall impulse response?

$$h = (h_1 * h_2) + h_3$$

table 2.1 Row 4

$$= \frac{e^{-3t} - e^{-5t}}{-3+5}u(t) + e^{-9t}\cos(3\pi t)u(t)$$

$$= \underline{\frac{1}{2}\left(e^{-3t} - e^{-5t}\right)u(t) + e^{-9t}\cos(3\pi t)u(t)}$$