Applications to Communications

- There are an almost infinite number of applications of 2704 content, but I wanted to pick just one to develop in some detail.

- Basic Analog Communications using Amplitude Modulation.

Recall the modulation theorem if the Fourier transform,

\[ x_1(t) x_2(t) \leftrightarrow \frac{1}{2\pi} \mathcal{F} \{ x_1(w) * x_2(w) \} \]

Suppose \( x_2(t) = \cos(w_c t) \), \( w_c = \) carrier frequency.

\[ x_1(t) \cos(w_c t) \leftrightarrow \frac{1}{2\pi} \mathcal{F} \{ x_1(w) * [\pi \delta(w-w_c) + \pi \delta(w+w_c)] \} \]

\[ \mathcal{F} \{ x_1(w) \} \mathcal{F} \{ x_2(w) \} = \int_{-\infty}^{\infty} \mathcal{F} \{ x_1(w') \} \mathcal{F} \{ x_2(w-w') \} \, dw' \]

\[ = 2 \int_{-\infty}^{0} \mathcal{F} \{ x_1(w') \} \delta(w-w'-w_c) \, dw' \]

\[ \Rightarrow \mathcal{F} \{ x_1(t) \cos(w_c t) \} = \mathcal{F} \{ x_1(w-w_c) \} + \frac{1}{2} \mathcal{F} \{ x_1(w+w_c) \} \]

he + \( x_1(t) \) = \( m(t) \) the modulating (or message signal)

\[ \cos(w_c t) \]

\[ m(t) \rightarrow \bigotimes \rightarrow y(t) \]
Note what this means in the Fourier domain.

\[ |M(\omega)| \]

\[ \text{B is bandwidth in Hz.} \]

\[ \text{sidebands} \]

\[ \text{upper} \]

\[ \text{lower} \]

This is called **double sideband suppressed carrier modulation**.

- **Applications**: radio & frequency multiplexing.

- **How do we undo this?**: De-modulation.

Want to shift back to \( \omega = 0 \)

Note \( y(t) = m(t) \cos(\omega_c t) \)

\[ y(t) \cos(\omega_c t) = m(t) \cos^2(\omega_c t) \]

\[ = m(t) \left[ \frac{1}{2} + \cos(2\omega_c t) \right] \]

\[ = \frac{1}{2} m(t) + m(t) \cos(2\omega_c t) \]

In the frequency domain,

\[ \text{lowpass filter} \]
To get rid of the unwanted side bands we can use a low pass filter

\[ m(t) \xrightarrow{\times} y(t) \xrightarrow{\oplus} \frac{1}{2} m(t) \]

Note an important constraint: \( W_c \geq 2 \pi f_B \) or overlap occurs.
Also \( m(t) \) must be bandlimited, so often there is a pre-filter to ensure this.

You might have noticed a technical problem here, the modulating and demodulating cosines must be in phase, or synchronous.

Often one end (transmitter) can also send the carrier, this allows the receiver to be quite simple. This is how AM radio works.

\[ y(t) = A \cos(w_c t) + m(t) \cos(w_c t) \]

where \( A + m(t) > 0 \neq t \)
Now, our receiver needs to detect the envelope of \( y(t) \).