* Applications of Fourier Transform: Filtering.

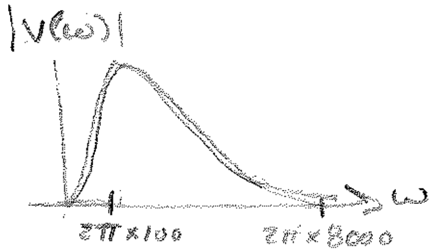
- From last time.

$$\begin{array}{ccc} x(t) \longrightarrow & \boxed{h(t)} & \longrightarrow y(t) \\ \mathcal{F}\downarrow & \mathcal{F}\downarrow & \updownarrow \\ X(\omega) \longrightarrow & \boxed{H(\omega)} & \longrightarrow Y(\omega) \end{array}$$

$$Y(\omega) = H(\omega)\,X(\omega) = \underbrace{|H(\omega)|\,|X(\omega)|}_{\text{frequency shaping.}}\, e^{j\angle H(\omega) + \angle X(\omega)}$$

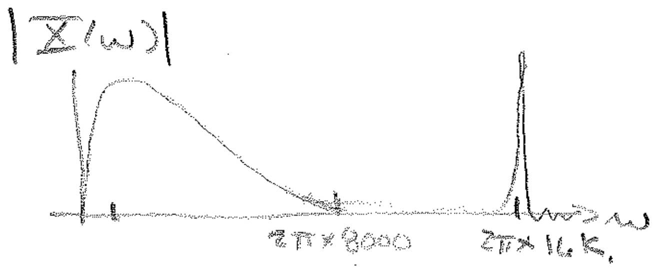
- The $|H(\omega)|$ can be viewed as shaping the original frequency spectrum of $X(\omega)$, and has many applications

- Application example: Removing Noise in an audio signal.
  - suppose we have audio from a voice. $v(t)$



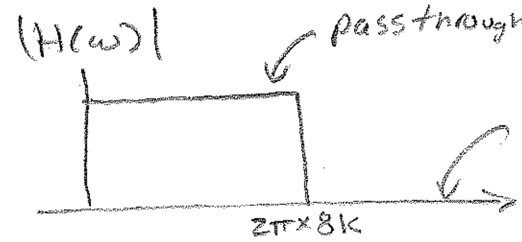
  - that is corrupted by some high pitched sound.

$$x(t) = v(t) + \sin(2\pi(16{,}000)t)$$



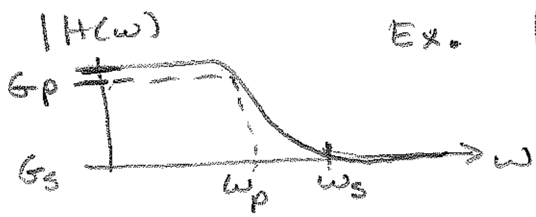
- How could we remove the corruption?
  - Shape $X(\omega)$ to look more like $V(\omega)$ using $H(\omega)$

- Example cont.



This is the ideal Low-pass filter.

- The ideal lowpass filter cannot be realized Instead.



Ex. $|H(\omega)|^2 = \dfrac{G_p}{1 + j\left(\frac{\omega}{\omega_p}\right)^{2n}}$

order n.

- we want $G_p \approx -3dB$ $\quad \omega_p \approx 2\pi \times 10K$

$G_s \approx -40dB$ $\quad \omega_s \approx 2\pi \times 16K.$

$-3dB = 10^{-3/20} = 0.708$

$-20dB = 10^{-20/20} = 0.1$

$-40dB = 10^{-40/20} = 0.01$

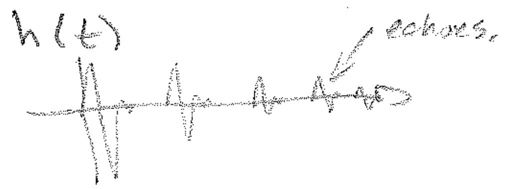
$\boxed{\text{Filter Wizard DEMO}}$

- The resulting circuit has the transfer function (Frequency Response) specified

The output reduces the signal component from the 16K corruption to

$$y(t) \approx v(t) + \underline{0.01} \sin(2\pi \times 16K\ t)$$

- Application Example #2: Simulation.
  - Given a venue for music, say a cathedral or concert hall
  - how might a music signal sound in that venue?
- Experiment: Record sound of a shot



- then given $X(\omega)$ of the song.

$$Y(\omega) = H(\omega)\, X(\omega)$$

$H(\omega)$ shapes the spectrum to the venues accousttics.

$\mathcal{F}^{-1}$

$y(t) \rightarrow$ play as it might sound in that venue.