* Filter Design Basics.

- Recall the frequency response determines the output due to an input sinusoid at a given frequency ω.

- It is common to design a transfer function, and thus a frequency response to affect sinusoids at different frequencies, differently.

Amplify. $\cos(\omega t) \rightarrow \boxed{H(\omega)} \rightarrow A\cos(\omega t + B)$

$A > 1 \quad B \approx 0$ (ideally)

Attenuate (Suppress) $\cos(\omega t) \rightarrow \boxed{H(\omega)} \rightarrow A\cos(\omega t + B)$

$A < 0 \quad B \approx 0$ (ideally)

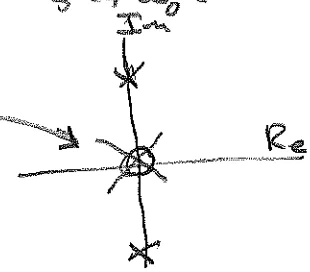
- This is done by placing poles (roots of $Q(s)$) and/or. zero's (roots of $P(s)$). We design polynomials.

* Effects of poles and zero's.

- Simple example: $\cos(\omega_0 t)u(t) \rightarrow \boxed{?} \rightarrow \sin(\omega_0 t)u(t)$

In Laplace Domain: $\frac{s}{s^2+\omega_0^2} \rightarrow \boxed{H(s)} \rightarrow \frac{\omega_0}{s^2+\omega_0^2}$

$H(s) = \frac{Y(s)}{X(s)} = \frac{\omega_0}{s}$, an integrator.



- We can also view this as a pole-zero cancellation.

- This gives us some intuition about how to amplify or supress.

Suppose $x(t) = \cos(2\pi t)u(t) + \cos(4\pi t)u(t)$

We want to e.g. Amplify $\cos(2\pi t)$ but Attenuate $\cos(4\pi t)$

Steady State: $\cos(2\pi t) \rightarrow \boxed{H(s)} \rightarrow A\cos(2\pi t + B)$ $\quad A > 1$

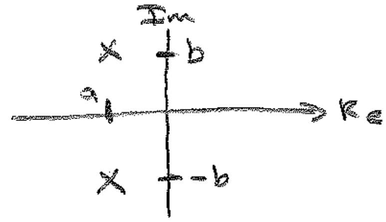
$C < 1$

$\cos(4\pi t) \rightarrow \boxed{H(s)} \rightarrow C\cos(4\pi t + D)$ $\quad B, D$ as small as possible.

Note: $A = |H(2\pi)|$ and $C = |H(4\pi)|$

Goal: Design $H(s)$ such that $|H(2\pi)|$ large while $|H(4\pi)|$ small.

- Effect of poles on $\cos(2\pi t)u(t)$



$$H(s) = \frac{K}{(s+a+jb)(s+a-jb)}$$

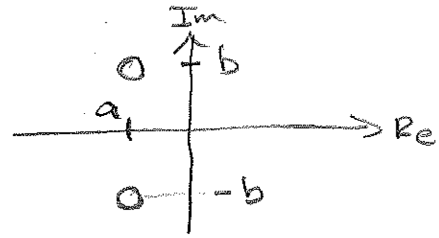
$$= \frac{K}{s^2+2as+a^2+b^2}$$

$$X(s) = \frac{s}{s^2+(2\pi)^2}$$

Then $Y(s) = H(s)X(s) = \dfrac{Ks}{(s^2+(2\pi)^2)(s^2+2as+a^2+b^2)}$

$\boxed{\text{DEMO}}$: Let $a>0$, $b=2\pi$
Then let $a<0$, unstable
Then let $a=0$, $b=\pi$
Then let $b \to 2\pi$, resonance.

- Effect of zeros on $\cos(2\pi t)u(t)$



$$H(s) = (s+a+jb)(s+a-jb)$$

$$= s^2+2as+a^2+b^2$$

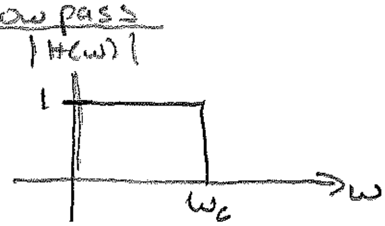
$$Y(s) = \frac{s(s^2+2as+a^2+b^2)}{s^2+(2\pi)^2}$$

$\boxed{\text{DEMO}}$: Let $a>0$ $b=2\pi$
then let $a \to 0$
Then switch to $a=0$, $b=\pi$ Let $b \to 2\pi$

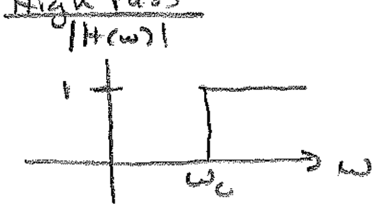
- In summary poles can be used to Amplify and zeros (with poles) can be used to attenuate.

- Ideal Filters. (all phases are ideally zero).

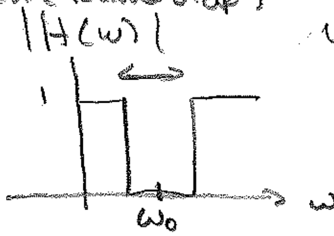
Low pass



High Pass

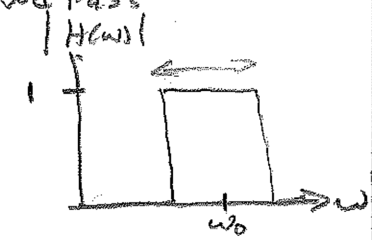


Notch (Band stop)



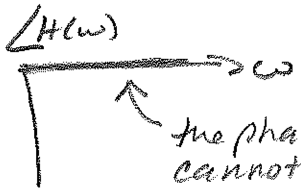
ω_0 = center frequency.

Band Pass

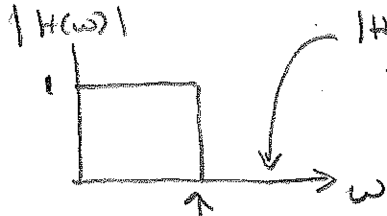


- Ideal Filters cannot be realized.

Example: Low-pass



the phase cannot be zero.

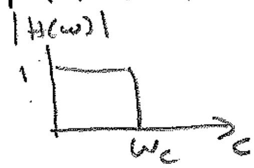


$|H(\omega)| \neq 0$ in any finite range of frequencies (Paley-Wiener)

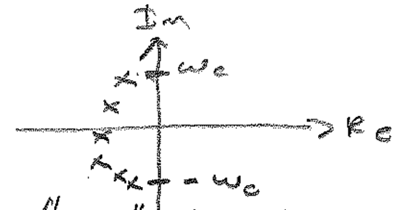
the transition cannot be infinitly steep.

- Practical Filters.

Low-Pass



$\Rightarrow$



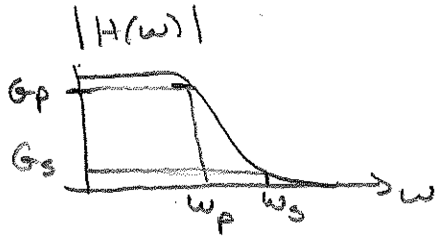
"wall" of poles
placement = family
(chebeshev, Butterworth)

- Nuber of poles is the <u>order</u> of the filter.

$\boxed{\text{DEMO}}$ - place poles
- Note response shape
- Note phase and DC Gain.

- To determine the order and exact shape we relax the ideal Filter.

LowPass (Non-Ideal)



G_s = Stop-band gain at ω_s

G_p = Pass-band gain at ω_p

For a Butterworth Filter the "wall of poles" is the polynomial,

$$|H(\omega)|^2 = \frac{G_p}{1 + j\left(\frac{\omega}{\omega_p}\right)^{2n}}$$

n = order of filter.

G_p = DC Gain

The correspondy Trasfer function is

$$H(s) = \frac{G_p}{\prod_{k=1}^{n} \frac{(s - s_k)}{\omega_p}} \qquad s_k = \omega_p e^{j\frac{(2k+n-1)\pi}{2n}} \qquad k = 1, 2, \dots n$$

n poles.

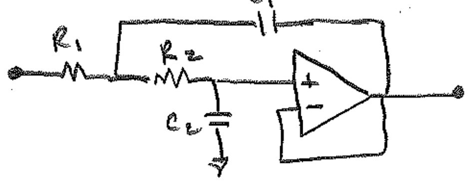
The larger n is the steeper the transition between frequencies.

- To reduce circuit complexity and sensitivity to R, C values filters are implemented in stages.

  - Two basic stages

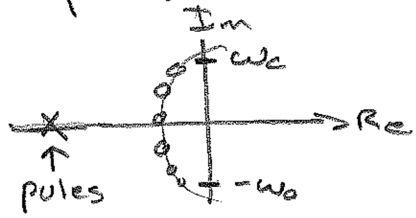
    n=1 (1st order) Butterworth

2nd order Butterworth using Sallen-Key Topology.



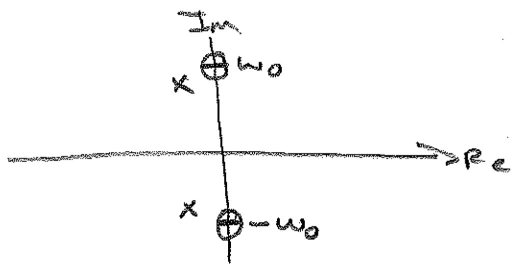
- For other filter types:

High Pass
Rarely used.



"wall of zeros" + pole.

Notch



Band Pass