Reading Bode Plots.

- Recall if a system is stable, then its frequency response:

\[ H(s) \bigg|_{s=j\omega} = H(j\omega) = H(\omega) \]

It tells us the steady-state response of sinusoidal inputs.

\[ A \cos(\omega t + \phi) \Rightarrow H(\omega) \Rightarrow A |H(\omega)| \cos(\omega t + \phi + \angle H(\omega)) \]

Thus a plot of \( |H(\omega)| \) and \( \angle H(\omega) \) tells us at a glance about the system's steady-state response.

Example: \( H(s) = \frac{1000}{s+1000} \) Stable?

\[ H(j\omega) = \frac{1}{j\omega + 1000} \]

\[ |H(j\omega)| = \frac{1000}{\sqrt{\omega^2 + 1000^2}} \]

\[ \angle H(j\omega) = \tan^{-1} \left( \frac{-\omega}{1000} \right) \]

Suppose the input is \( 10 \sin(100t) \cos(t) \)

\[ 10 \sin(100t) = 10 \cos(100t - \frac{\pi}{2}) \]

The steady-state output \( t > 0 \) is

\[ y_{ss}(t) = 10 \left| \frac{1000}{100^2 + 1000^2} \right| \cos(100t - \frac{\pi}{2} + \tan^{-1}(\frac{100}{1000})) \]

\[ \approx 9.95 \sin(100t - 0.1) \]

\[ \uparrow \text{Small decrease in magnitude} \quad \uparrow \text{Small phase shift} \]
So, what happens if we plot a typical frequency response. E.g., compare \( \frac{1}{s+1} \) and \( \frac{1}{(s+1)(s+100)} \)

- **DEMO**
  - Linear-Linear Plot
  - Log-Linear Plot
  - Log-Log Plot.

This shows us the advantage of plotting \( |H(j\omega)| \) on a Log-Log scale and \( \angle H(j\omega) \) on a Log-Linear scale.

A further refinement is that in signal processing we are often interested in differences detectable by humans, which sound seems twice as loud.

Psychophysics tells us humans perceive differences on a log, scale. (Weber-Fechner Law)

By convention the y-axis of a frequency response plot is such that a slope of \( 20 \) corresponds to a \( 10 \)-fold increase in power. A slope of \(-20\) then is a \( 10 \)-fold decrease in power.

Together this gives us the **Bode Plot**:
- a log-linear plot of \( 20 \log_10 |H(j\omega)| \) (units dB)
- a log-linear plot of \( \angle H(j\omega) \) degrees \( \pm \) radians.

Traditionally, Bode Plots made it easy to draw frequency response plots by hand.

Now it is easier to do by computer: Mathematica, Matlab.

In 3704 you will learn how to draw them by hand, my concern is that you can read them.
Procedure to use a Bode Plot to determine the steady-state response to sinusoids.

1. Locate the point in the magnitude and phase plots that correspond to the frequency of interest.

\[ |H(\omega)| \text{ dB} \quad \omega \quad \text{w (rad/s)} \]

2. Convert from dB to gain.
   \[ A = 20 \log_{10} |H(\omega)| \quad \text{so} \quad |H(\omega)| = 10 \]

3. Then you can construct the steady-state response
   \[ |H(\omega)| \cos (\omega t + \angle H(\omega)) \]