★ Reading Bode Plots.

- Recall *if* a system is stable, then its frequency response.

$$H(s)\Big|_{s=j\omega} = H(j\omega) = H(\omega)$$

tells us the steady-state response of sinusoidal inputs.

$$A\cos(\omega t + \phi) \longrightarrow \boxed{H(\omega)} \longrightarrow A|H(\omega)|\cos(\omega t + \phi + \angle H(\omega))$$

Thus a plot of $|H(\omega)|$ and $\angle H(\omega)$ tells us at a glance about the systems steady state response.

Example: $H(s) = \dfrac{1000}{s+1000}$ stable?

$$H(\omega) = \frac{1}{j\omega + 1000} \Rightarrow |H(\omega)| = \frac{1000}{(\omega^2 + 1000^2)^{1/2}}$$

$$= 1000\left[\frac{1000 - j\omega}{1000^2 + \omega^2}\right] \qquad \angle H(\omega) = \tan^{-1}\left(\frac{-\omega}{1000}\right)$$

Suppose the input is $10\sin(100t)u(t)$

$$10\sin(100t) = 10\cos(100t - \pi/2)$$

the steady state output ($t >> 0$ is)

$$y_{ss}(t) = 10\left|\frac{1000}{100^2 + 1000^2}\right| \cos\left(100t - \pi/2 + \tan^{-1}\left(\frac{-100}{1000}\right)\right)$$

$$\approx 9.95\sin(100t - 0.1)$$

small decrease in magnitude

small phase shift.

- So, what happens if we plot a typical Frequency Response.

  E.g. Compare $\frac{1}{s+1}$ and $\frac{1}{(s+1)(s+100)}$

  $\boxed{\text{DEMO}}$ — Linear - Linear Plot
  — Log - Linear Plot
  — Log - Log Plot.

- This shows us the advantage of plotting

  $|H(\omega)|$ on a Log-Log scale

  $\angle H(\omega)$ on a Log-linear scale.

- A further refinement is that in signal processing we are often interested in differences detectable by humans.

  $\boxed{\text{DEMO}}$ Which sound seems twice as loud.

- Psychophysics tells us humans perceive differences on a $\log_{10}$ scale. (Weber-Fechner Law)

  By <u>convention</u> the y-axis of a frequency response plot is such that a slope of 20 corresponds to a 10-fold <u>increase</u> in <u>power</u>. A slope of -20 then is a 10-fold <u>decrease</u> in <u>power</u>.

- Together this gives us the <u>Bode Plot</u>.
  - a log-linear plot of $20 \log_{10} |H(\omega)|$ (units dB)
    decibels
  - a log-linear plot of $\angle H(\omega)$ degrees <u>or</u> radians.

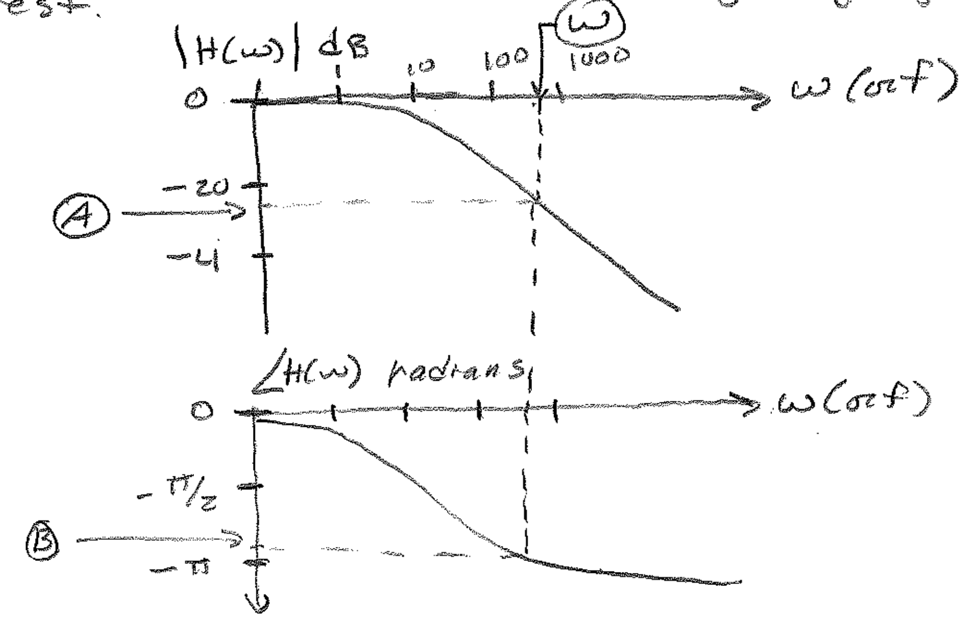
- Traditionally, Bode-Plots made it easy to draw frequency response plots by hand.

  Now it is easier to do by computer: — Mathematica
  — Matlab.

- In 3704 you will learn how to draw them by hand, my concern is that you can <u>read</u> them.

- Procedure to use a Bode Plot to determine the steady-state response to sinusoids.

1. Locate the point in the magnitude and phase plots that correspond to the frequency of interest.



2. Convert from dB to Gain.

$$\text{Ⓐ} = 20\log_{10}|H(\omega)| \quad \text{so} \quad |H(\omega)| = 10^{\text{Ⓐ}/20}$$

Ⓑ if in degrees convert to radians.

3. Then you can constuct the steady state response

$$|H(\omega)|\cos(\omega t + \angle H(\omega))$$