Recall if a system is stable then the frequency response

\[ H(s) \bigg|_{s=j\omega} = H(j\omega) = H(\omega) \]

tells us the steady-state response of sinusoidal inputs

\[ A \cos(\omega t + \phi) \rightarrow H(\omega) \rightarrow A |H(\omega)| \cos(\omega t + \phi + \Delta(\omega)) \]

*since \( \sin(\omega t) = \cos(\omega t - \frac{\pi}{2}) \) then

\[ A \sin(\omega t + \phi) \rightarrow H(\omega) \rightarrow A |H(\omega)| \sin(\omega t + \phi + \Delta(\omega)) \]

Thus a plot of \( |H(\omega)| \) and \( \Delta(\omega) \) tells us a lot at a glance about the system's steady-state response.

So what happens if we plot a typical freq response

\[ \frac{1}{s+1} \quad \text{and} \quad \frac{1}{(s+1)(s+10)} \]


This shows there is an advantage to plotting \( |H(\omega)| \) on a log-log scale

\( \Delta(\omega) \) on a linear-log scale
- A further refinement is that for signal processing we are often interested in differences detectable by humans. Weber–Fechner law
  - Play a tone with given amplitude.
  - Then play with double amplitude.
  - Then play with 10x amplitude. Ask which ones were twice as loud, as first.
- We perceive differences on a log₁₀ scale (approx).
  \[ x(t) = A \cos(600\pi t) \]
  \[ 2A \text{ vs. } 10A \]
  This makes a dB scale advantageous.
  \[ 20 \text{ vs. } 10 \text{ fold decrease in power} \]

By convention the y-axis of a frequency response is such that a slope of 20 corresponds to a 10-fold increase in power; -20 to 10-fold decrease in power.

- Together the above gives us the Bode Plot:
  - A log-linear plot of 20 log |H(w)| (units dB)
  - A log-linear plot of \( \angle \)H(w) (in degrees or rad).
- The x-axis may be in Hz or rad/s.

- Traditionally Bode Plots made it easy to draw frequency domain plots. Now that is easy to do on a computer.

Mathlab Example:

```
loglog(w, squeeze(w))
semilogx(w, squeeze(w))
mag, phase = bode(H, Num, Den)
```

Mention quirks
So to use a Bode plot to determine the steady state response:

- First locate the point in the magnitude and phase plots that correspond to the frequency of interest, $\omega$.

- Read of values for $A$ and convert from dB:
  \[ A = 20 \log_{10} |H(\omega)| \]
  \[ A = 20 \% \]

- Read of values for phase $\phi$.
  (If in degrees convert to rad.)
  \[ \phi = \phi \]

- Then you can construct the response:
  \[ |H(\omega)| \cos(\omega t + \phi(\omega)) \]