★ System Realization

Recall from last time block diagrams can act as an intermediate step in the realization or synthesis of a particular transfer function.

– For now, we gloss over how to design a transfer function, but focus on implementing it in a circuit.

– Suppose we are given the following TF and are told to build a circuit that implements it.

$$H(s) = \frac{s+2}{s^2+9s+20}$$

• First, lets divide through top + bottom by the highest power of s.

$$H(s) = \frac{\frac{1}{s} + \frac{2}{s^2}}{1 + \frac{9}{s} + \frac{20}{s^2}} = \underbrace{\left[\frac{1}{s} + \frac{2}{s^2}\right]}_{H_1(s)} \underbrace{\left[\frac{1}{1 + \frac{9}{s} + \frac{20}{s^2}}\right]}_{H_2(s)}$$

• This can be implemented by a serial cascade.

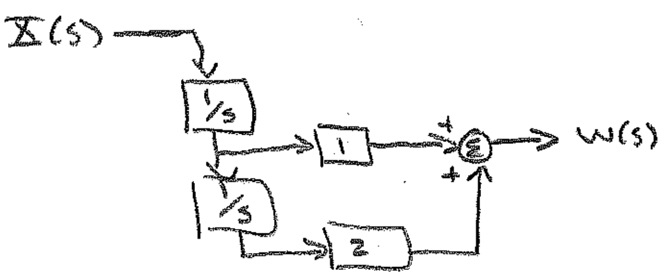
$$X(s) \rightarrow \boxed{H_1} \xrightarrow{W(s)} \boxed{H_2} \rightarrow Y(s)$$

• Lets break H_1 down further using series connections:

$$W(s) = \frac{1}{s} X(s) + \frac{2}{s^2} X(s)$$

$$= \underbrace{\frac{1}{s} X(s)}_{} + \frac{2}{s}\Big(\underbrace{\frac{1}{s} X(s)}_{}\Big)$$

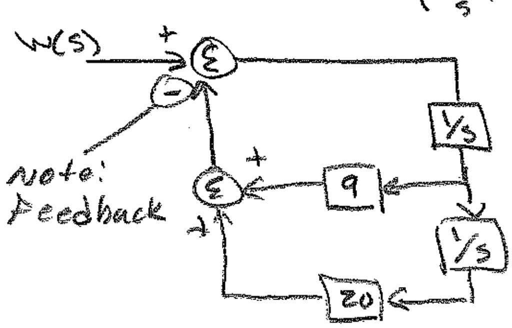
repeated



• Now for H_2, note $Y(s) = H_2(s) W(s)$ and H_2 is in the form of feedback.

$$W(s) = \left(1 + \frac{9}{s} + \frac{20}{s^2}\right) Y(s) = Y(s) + \left(\frac{9}{s} + \frac{20}{s^2}\right) Y(s)$$

or $$Y(s) = W(s) - \left(\frac{9}{s} + \frac{20}{s^2}\right) Y(s)$$



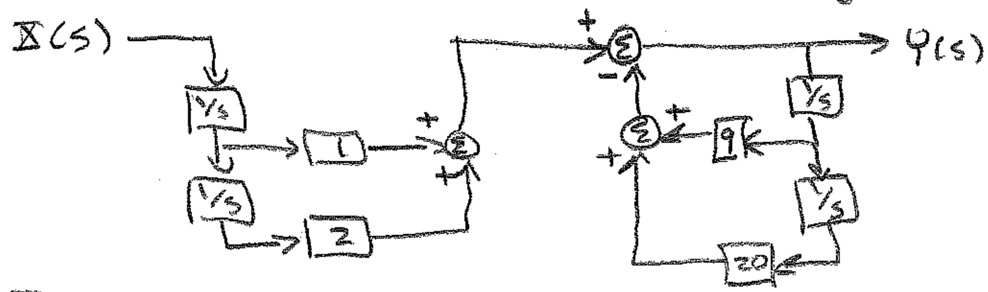
Feedback Motif

$$\frac{G(s)}{1 + G(s)H(s)}$$

$G(s) = 1$

$H(s)$ is series $\frac{9}{s} + \frac{20}{s^2}$

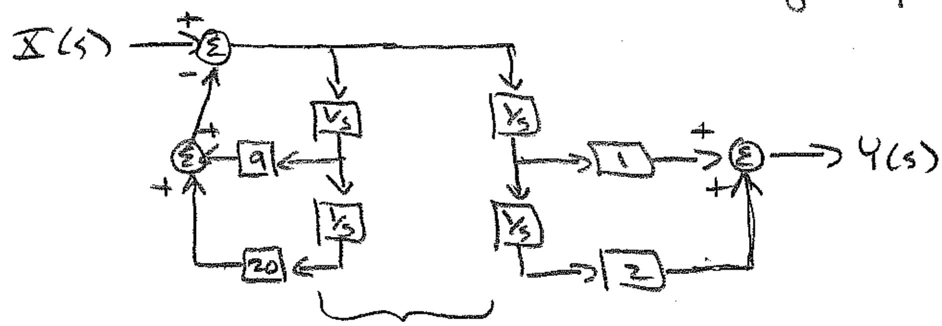
- Putting the individual BD's back together.



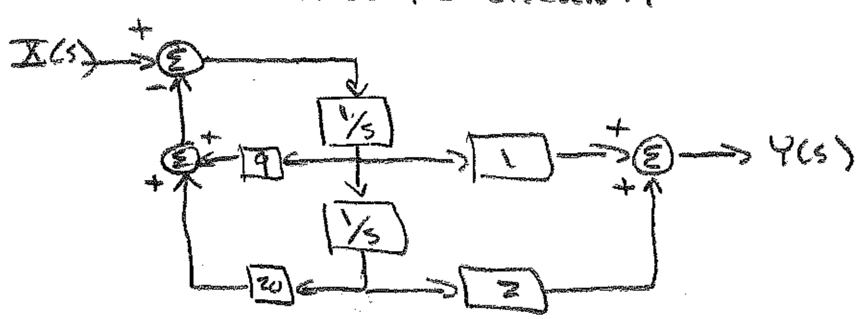
- This is the Direct Form I realization of the TF. Note it consists of just 3 types of blocks

$\rightarrow\boxed{1/s}\rightarrow$, $\rightarrow\boxed{K}\rightarrow$, Σ

- Recall that if $H(s) = H_1(s)H_2(s) = H_2(s)H_1(s)$ thus we can reverse the order of H_1 + H_2



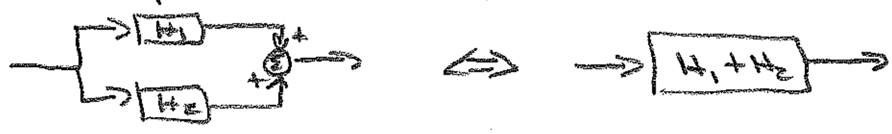
Note, these two paths are identical thus redundant.



This is called the direct Form II (Canonical) realization of the TF.

- Direct Form I: 4 $\boxed{s^{-1}}$, 4 $\boxed{K}$,
- Direct Form II: 2 $\boxed{s^{-1}}$, 4 $\boxed{K}$. 2 fewer integrators.

- Recall the parallel motif.



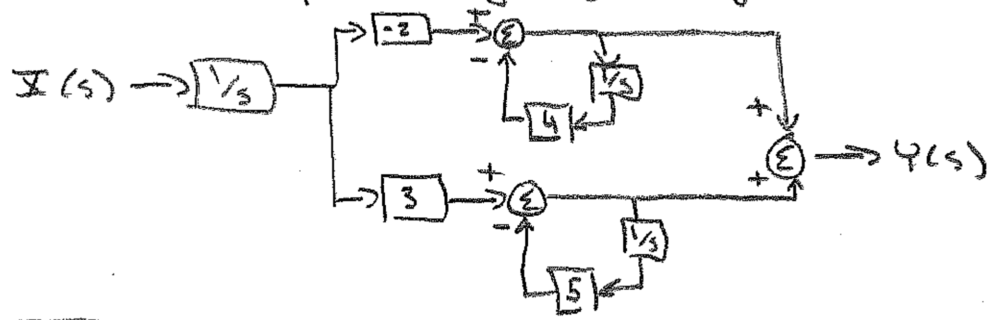
- Lets do a PFE of our example:

$$H(s) = \frac{s+2}{s^2+9s+20} = \frac{A}{s+4} + \frac{B}{s+5} = \underbrace{\frac{-2}{s+4}}_{H_1(s)} + \underbrace{\frac{3}{s+5}}_{H_2(s)}$$

- Direct Form of $H_1 = \dfrac{-2/s}{1 + 4/s}$

- Direct Form of $H_2 = \dfrac{3/s}{1 + 5/s}$

- Combining these in parallel gives (using Direct Form I)



- Which Form one should use depends on which overall uses the fewest components with close to standard values, but one case that does not work is 2^{nd} order complex roots and parallel motif.

Example: $H(s) = \dfrac{1}{s^2+4s+8} = \dfrac{A}{(s+2+j2)} + \dfrac{B}{(s+2-j2)}$

$$= \frac{\frac{1}{4}j}{(s+2+j2)} + \frac{-\frac{1}{4}j}{(s+2-j2)}$$

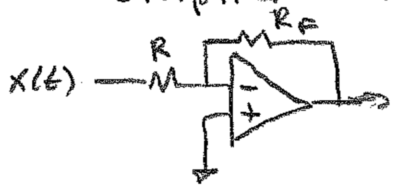
- Implementing this requires <u>complex</u> adders and multipliers, which do not exist in continuous circuits.

- Note this example can be implemented using a Direct Form (I or II) using real values.

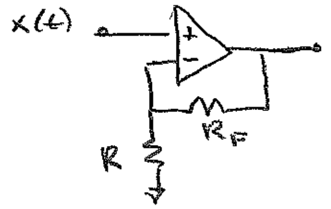
$$H(s) = \underbrace{\left(\frac{1}{s^2}\right)\left(\frac{1}{1+\frac{4}{s}+\frac{8}{s^2}}\right)}_{DFI} = \underbrace{\left(\frac{1}{1+\frac{4}{s}+\frac{8}{s^2}}\right)\left(\frac{1}{s^2}\right)}_{DFII}$$

– So, how do we go from combinations of s^{-1}, K, Σ to a circuit?

– Multiplier $\rightarrow \boxed{K} \rightarrow$

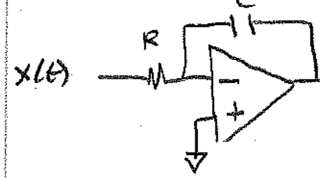


$$y(t) = -\frac{R_F}{R} x(t) \overset{\mathcal{L}}{\Rightarrow} Y(s) = -\frac{R_F}{R} X(s)$$
$$= K\, X(s),\ K<0$$



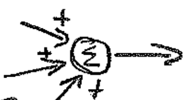
$$y(t) = \frac{R+R_F}{R_F} x(t) \overset{\mathcal{L}}{\Rightarrow} Y(s) = \frac{R+R_F}{R_F} X(s)$$
$$= K\, X(s) \quad K>1$$

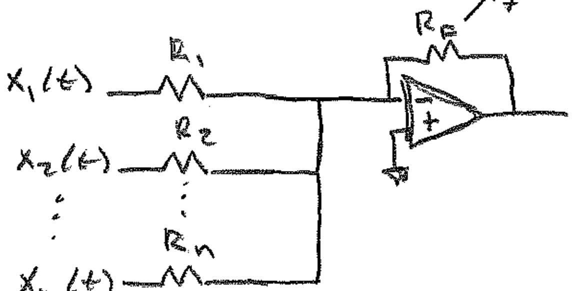
– Integrator $\rightarrow \boxed{s^{-1}} \rightarrow$



$$y(t) = -\frac{1}{RC}\int_0^t x(\tau)\,d\tau \overset{\mathcal{L}}{\Rightarrow} Y(s) = -\frac{1}{RC}\cdot\frac{1}{s}$$
$$= \frac{K}{s},\ K<0$$

– Summer / Adder





$$y(t) = \sum_{i=1}^{n} -\frac{R_F}{R_i} x_i(t)$$

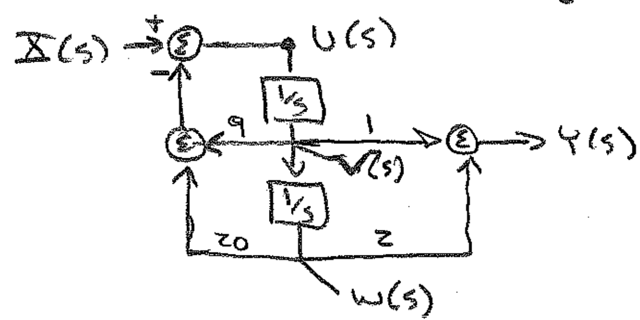
$$\Downarrow \mathcal{L}$$

$$Y(s) = \sum_{i=1}^{n} -\frac{R_F}{R_i} X_i(s)$$
$$= \sum_{i=1}^{n} K_i X_i(s),\ K_i<0$$

– There are some practical issues, getting the signs to match up, constraints on R, C values, require tunning resistors (potentiometers), variations in component tolerances, etc.

– This becomes a constrained optimization problem.

- Example: Consider our original example using PF II.



$$U(s) = X(s) - 9\,V(s) - 20\,W(s)$$

$$V(s) = \frac{1}{s}\,U(s)$$

$$W(s) = \frac{1}{s}\,V(s)$$

$$Y(s) = V(s) + 2\,W(s)$$

- How may degrees of freedom do we have?

$$Y(s) = K_1\,V(s) + K_2\,W(s)$$

$$V(s) = \frac{K_3}{s}\,U(s)$$

$$W(s) = \frac{K_4}{s}\,V(s)$$

$$U(s) = K_1\,X(s) + K_6\,V(s) + K_7\,W(s)$$

Re-assembly the TF

$$\frac{Y(s)}{U(s)} = K_1K_3\frac{1}{s} + K_2K_3K_4\frac{1}{s^2}$$

$$\frac{U(s)}{X(s)} = \frac{K_5}{1 - K_6K_3\frac{1}{s} - K_7K_3K_4\frac{1}{s^2}}$$

So, we want $K_1K_3 = 1$, $K_2K_3K_4 = 2$

$K_5 = 1$, $K_6K_3 = -9$, $K_7K_3K_4 = -20$

AND ideally $K_1, K_2, \cdots K_7 < 0$ (not possible)

- Note! These numerical values are wildly unrealistic.