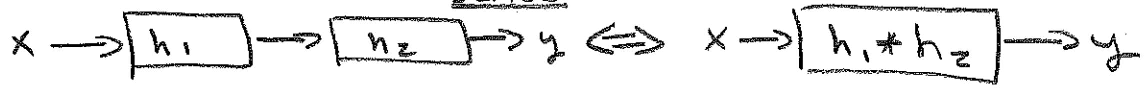
- Block Diagrams.

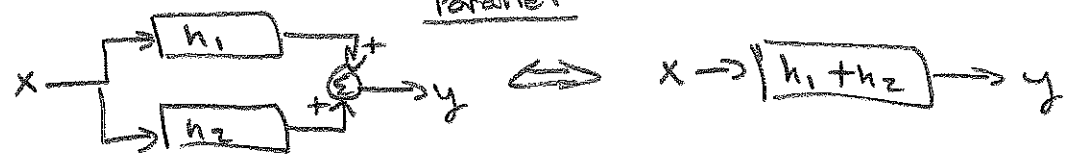
  - Recall from the time domain the motifs

Series

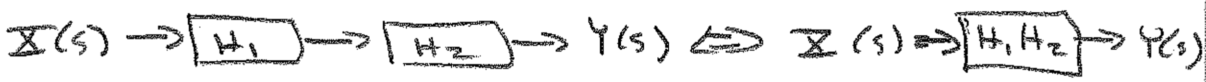


and

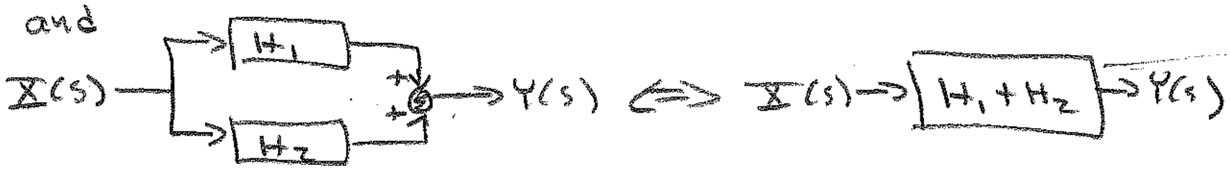
Parallel



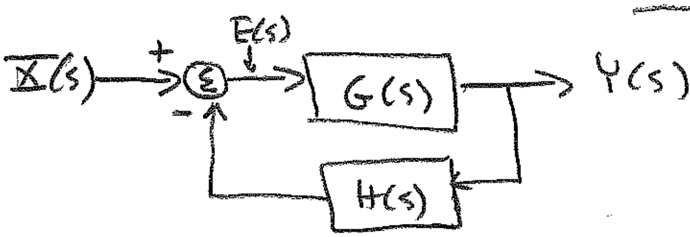
  - In the Laplace Domain these become.



and



- A third motif that is easy to represent in Laplace but not in time domain is feedback.



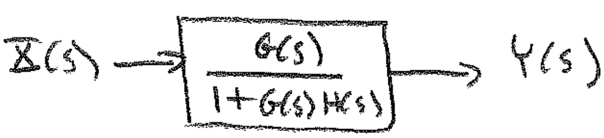
Let $E(s) = X(s) - H(s)Y(s)$

and $Y(s) = E(s)G(s)$

then $Y(s) = G(s)X(s) - G(s)H(s)Y(s)$

or $(1 + G(s)H(s))Y(s) = G(s)X(s)$

$$Y(s) = \underbrace{\frac{G(s)}{1 + G(s)H(s)}}_{\text{overall transfer function}} X(s)$$

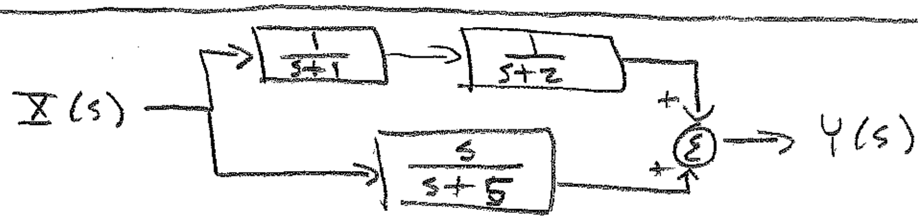


- These basic building blocks give us the ability to visualize systems of larger complexity using <u>abstraction</u>.

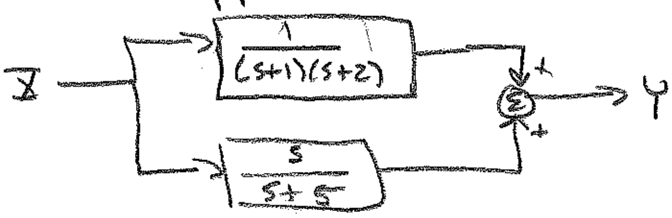
- There are two usefull skills for manipulating and using block diagrams.
  - Given a block Diagram, determine the overall transfer function.
  - Given a Transfer function <u>synthesize</u> (design) a block diagram.

- Note this is not a one-to-one correspondance there are many block diagrams that can be equivalent to the same transfer function.

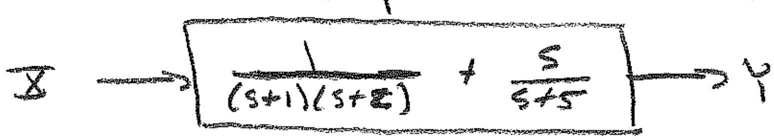
Example:



- First combine upper branch in series.



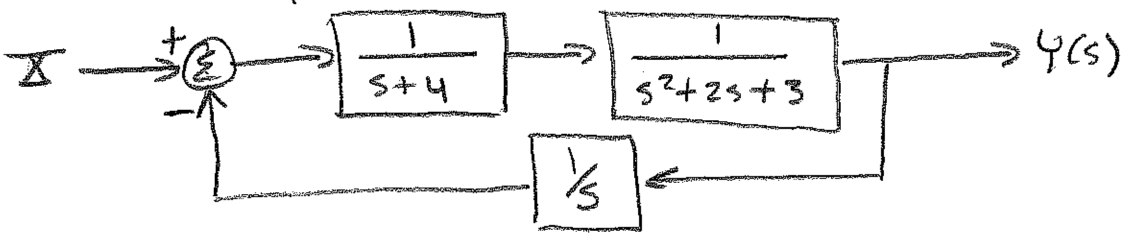
- Then combine the parallel branches.



- Putting it into standard form $H(s) = \frac{P(s)}{Q(s)}$ gives.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s^3 + 3s^2 + 3s + 5}{s^3 + 8s^2 + 17s + 10}$$

Another example:

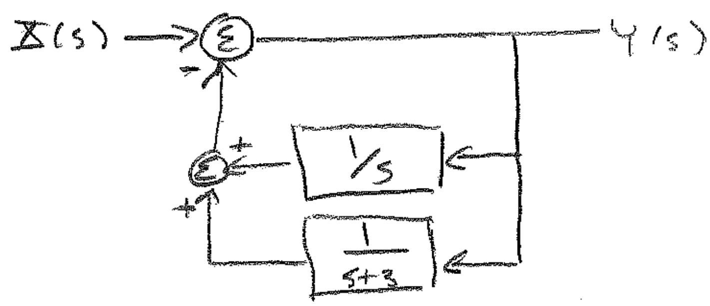


- combine in series $\frac{1}{(s+4)(s^2+2s+3)} = G(s)$

then use Feedback.
$$H(s) = \frac{\frac{1}{(s+4)(s^2+2s+3)}}{1 + \frac{1}{s(s+4)(s^2+2s+3)}}$$

Gives $H(s) = \frac{s}{s^4+6s^3+11s^2+12s+1}$

- Another Example:

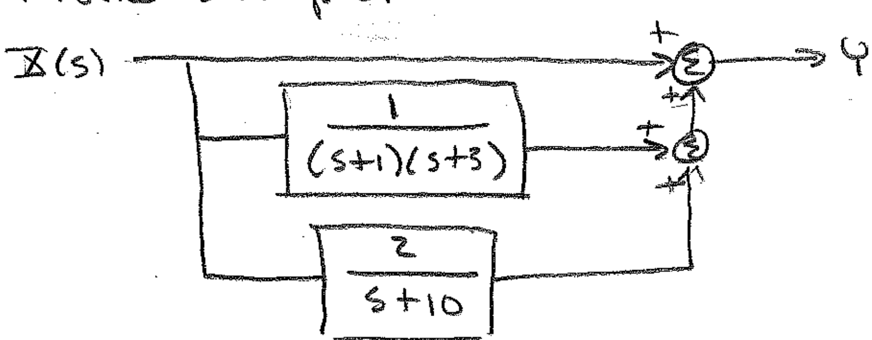


$G(s) = 1$

$H(s) = \frac{1}{s} + \frac{1}{s+3}$

$$\frac{G(s)}{1+G(s)H(s)} = \frac{1}{1+\frac{1}{s}+\frac{1}{s+3}} = \frac{s^2+3s}{s^2+5s+3}$$

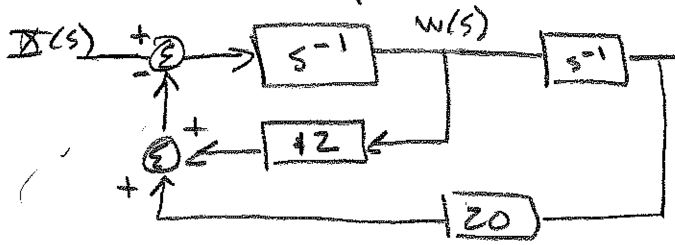
- Another Example!



$$\frac{Y(s)}{X(s)} = 1 + \frac{1}{(s+1)(s+3)} + \frac{2}{s+10}$$

- Now lets use our knowledge of block diagrams in analysis.

Given the following Block Diagram of a system, what is the impulse response (in the time domain)?



(1) $Y(s) = s^{-1}W(s)$

(2) $W(s) = s^{-1}(X - 12W - 20Y)$

solve (2) for W

$$W = s^{-1}X - 12s^{-1}W - 20s^{-1}Y$$

$$(1+12s^{-1})W = s^{-1}X - 20s^{-1}Y$$

$$(3)\quad W = \frac{s^{-1}}{1+12s^{-1}}X - \frac{20s^{-1}}{1+12s^{-1}}Y$$

put (3) → (1) solve for Y(s)

$$Y = \frac{s^{-2}}{1+12s^{-1}}X - \frac{20s^{-2}}{1+12s^{-1}}Y$$

$$(1+12s^{-1})Y + 20s^{-2}Y = s^{-2}X$$

$$(1+12s^{-1}+20s^{-2})Y = s^{-2}X$$

$$Y = \frac{s^{-2}}{1+12s^{-1}+20s^{-2}}X \Rightarrow H(s) = \frac{1}{s^2+12s+20}$$

- Do PFE $H(s) = \frac{A}{s+2} + \frac{B}{s+10}$

$$A = \left.\frac{1}{s+10}\right|_{s=-2} = \frac{1}{8} \qquad B = \left.\frac{1}{s+2}\right|_{s=-10} = \frac{1}{-8}$$

AND $h(t) = \left[\frac{1}{8}e^{-2t} - \frac{1}{8}e^{-10t}\right]u(t)$ using Table.

- Note, this could be represented as a parallel combination.

TOPS