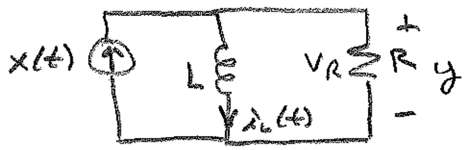
★ Application: Circuit Analysis using Laplace.

- Now that we can solve LCCDE using Laplace it is obvious that it could be applied to circuits.

Example:



$L=1 \quad R=2 \quad i_L(0^-)=0$

$x(t)=(1-e^{-t})u(t)$

KCL: $i_L + \frac{V_R}{R} = x$ and $V_L = V_R = L i_L'$

$$\frac{L}{R} i_L' + i_L = x \quad \text{or} \quad i_L' + \frac{R}{L} i_L = \frac{R}{L} x$$

substituting values $\quad i_L' + 2 i_L = 2x \qquad i_L(0^-)=0$

TAKE Laplace $\quad sI_L(s) + 2I_L(s) = 2X(s)$

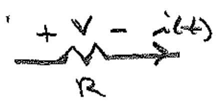
Solve for I_L $\quad I_L(s) = \frac{2}{s+2} X(s)$

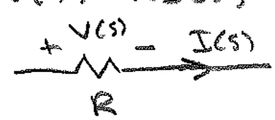
substitute $X(s) = \frac{1}{s} - \frac{1}{s+1} \Rightarrow I_L(s) = \frac{2}{s(s+2)} - \frac{2}{(s+1)(s+2)}$

$Y(s) = LsI_L(s) = \frac{2}{s+2} - \frac{2s}{(s+1)(s+2)}$ The solution in Laplace Domain.

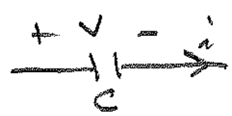
- We can save some effort by applying the Laplace transform First.

Resistor: $v(t) = Ri(t) \xleftrightarrow{\mathcal{L}} V(s) = RI(s)$





Capacitor: $i(t) = Cv'(t) \longleftrightarrow I(s) = CsV(s) - Cv(0^-)$

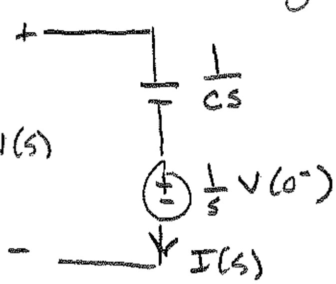


or equivalently

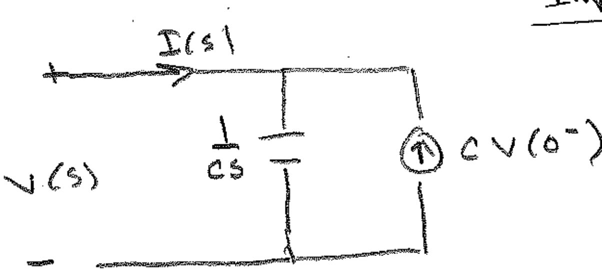
$V(s) = \frac{1}{Cs} I(s) + \frac{1}{s} v(0^-)$

Impedance

These are equivalent to the circuits

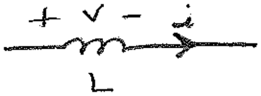


Use with KVL



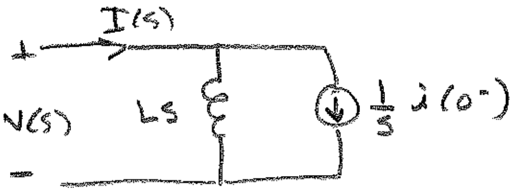
Use with KCL

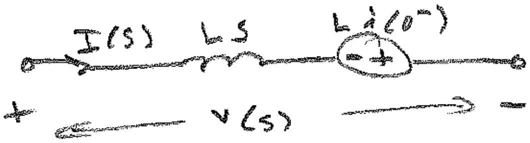
- Inductor. $v(t) = L\, i'(t)$ $\Longleftrightarrow$ $V(s) = \underbrace{Ls}_{\text{impedance.}} I(s) - L\, i(0^-)$



or
$I(s) = \frac{1}{Ls} V(s) + \frac{1}{s} i(0^-)$

Equivalent circuits.



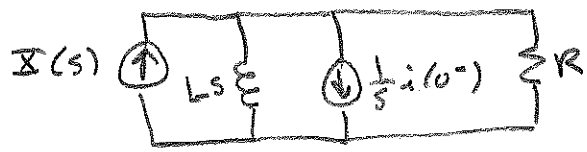


use with KCL

use with KVL.

Lets redo our example:

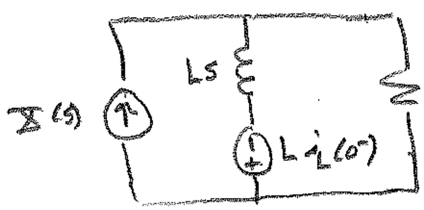
$\Longrightarrow$



model I

- The best model to use is model I since KCL looks like a good Approach.

OR



model II

- KCL with Model I

$$X(s) = \frac{Y(s)}{Ls} + \frac{1}{s} i(0^-) + \frac{Y(s)}{R}$$

multiply through by Rs $\Rightarrow$ $Rs\,X(s) = \frac{R}{L} Y(s) + \frac{R}{L} i(0^-) + s\,Y(s)$

rearranging $\left(s + \frac{R}{L}\right) Y(s) = Rs\,X(s) + \frac{R}{L} i_L(0^-)$

$$Y(s) = \frac{Rs}{s + \frac{R}{L}} X(s) + \frac{R}{L} i_L(0^-)$$

substituting values. $Y(s) = \frac{2s}{(s+2)} X(s)$

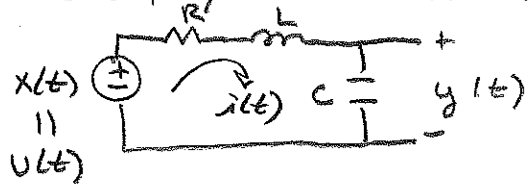
Substituting $X(s) = \frac{1}{s} - \frac{1}{s+1}$

$$Y(s) = \frac{2}{s+2} - \frac{2s}{(s+1)(s+2)}$$

The same result.

- Note: This approach is used heavily in AC Circuits where zero-state is usually the case and $x(t)$ are sinusoids.

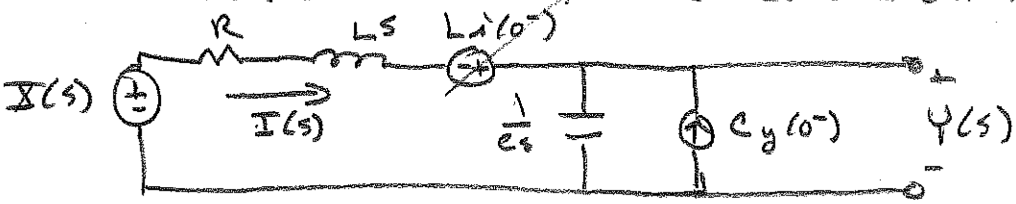
- Lets try another example. series RLC step response with I.C.



$R = 2 \quad L = \frac{1}{4} \quad C = \frac{1}{3}$

$i_L(0^-) = 0 \quad y(0^-) = 2$

Transform the circuit. (what are other choices?)



- KVL: $X(s) = RI(s) + LsI(s) + Y(s)$ (1)

- KCL: $I(s) = \dfrac{1}{\frac{1}{Cs}} Y(s) - Cy(0^-) = CsY(s) - Cy(0^-)$ (2)

- substituting (2) $\rightarrow$ (1)

$$X(s) = RCsY(s) - RCy(0^-) + LCs^2Y(s) - LCsy(0^-) + Y(s)$$

- collecting terms

$$LCs^2Y(s) + RCsY(s) + Y(s) = X(s) + RCy(0^-) + LCsy(0^-)$$

- divide through by LC

$$s^2Y(s) + \frac{R}{L}sY(s) + \frac{1}{LC}Y(s) = \frac{1}{LC}X(s) + \frac{R}{L}y(0^-) + sy(0^-)$$

or

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)Y(s) = \frac{1}{LC}X(s) + \left(s + \frac{R}{L}\right)y(0^-)$$

- Substituting values

$$(s^2 + 8s + 12)Y(s) = 12X(s) + 2(s+8)$$

or

$$Y(s) = \underbrace{\frac{12}{s^2+8s+12}X(s)}_{\text{zero-state response}} + \underbrace{\frac{2(s+8)}{s^2+8s+12}}_{\text{zero-input response}}$$

- when $x(t) = u(t)$

$$X(s) = \frac{1}{s}$$

- Example cont. Applying input, and $s^2+8s+12 = (s+2)(s+6)$

$$Y(s) = \frac{12}{s(s+2)(s+6)} + \frac{2(s+8)}{(s+2)(s+6)}$$

- To find $y(t)$ we need to take $\mathcal{L}^{-1}$ of $Y(s)$

• First Term: $\frac{12}{s(s+2)(s+6)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+6}$

Lets use the "coverup" method to find A, B, C.

$$A = \left.\frac{12\cancel{(s)}}{\cancel{s}(s+2)(s+6)}\right|_{s=0} = \frac{12}{12} = 1$$

$$B = \left.\frac{12\cancel{(s+2)}}{s\cancel{(s+2)}(s+6)}\right|_{s=-2} = \frac{12}{-2(4)} = \frac{12}{-8} = -\frac{3}{2}$$

$$C = \left.\frac{12\cancel{(s+6)}}{s(s+2)\cancel{(s+6)}}\right|_{s=-6} = \frac{12}{-6(-4)} = \frac{12}{24} = \frac{1}{2}.$$

• Second Term: $\frac{2(s+8)}{(s+2)(s+6)} = \frac{D}{s+2} + \frac{E}{s+6}$

$$D = \left.\frac{2(s+8)\cancel{(s+2)}}{\cancel{(s+2)}(s+6)}\right|_{s=-2} = \frac{2(6)}{4} = 3$$

$$E = \left.\frac{2(s+8)\cancel{(s+6)}}{(s+2)\cancel{(s+6)}}\right|_{s=-6} = \frac{2(2)}{-4} = -1$$

Thus $Y(s) = \frac{1}{s} + \frac{3-3/2}{s+2} + \frac{-1+\frac{1}{2}}{s+6} = \frac{1}{s} + \frac{3/2}{s+2} + \frac{-1/2}{s+6}$

using Table:

$$\underline{y(t) = \left(1 + \tfrac{3}{2}e^{-2t} - \tfrac{1}{2}e^{-6t}\right)u(t)}$$

★ Summary: This approach makes Circuit Analysis easier when in zero, state conditions, particular for steady-state analysis.