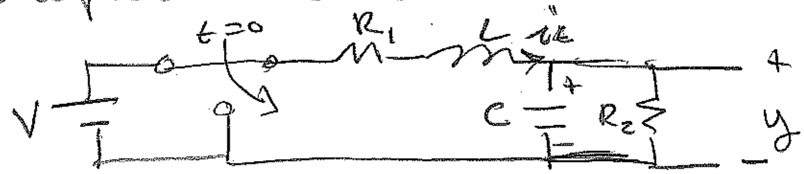
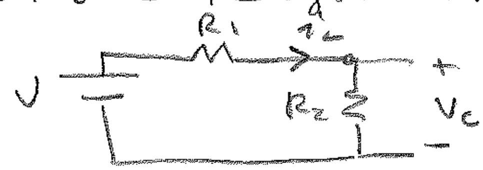
- By way of review of time-domain analysis lets do a practice exam.

1



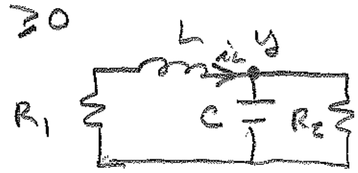
a) Determine the initial conditions on energy storage elements, i_L & V_c.

Before $t=0$ the system is under DC.



$$\boxed{V_c(0^-) = \frac{R_2}{R_1+R_2} V \qquad i_L(0^-) = \frac{V}{R_1+R_2}}$$

b) for $t \geq 0$



KCL: $i_L = Cy' + \frac{1}{R_2} y$ (1)

KVL: $R_1 i_L + L i_L' + y = 0$ (2)

Let (3) be derivative of (1) $i_L' = Cy'' + \frac{1}{R_2} y'$

Then substitute (1) & (3) into (2)

$$R_1 \left(Cy' + \frac{1}{R_2} y\right) + L\left(Cy'' + \frac{1}{R_2} y'\right) + y = 0$$

$$R_1 C y' + \frac{R_1}{R_2} y + LCy'' + \frac{L}{R_2} y' + y = 0$$

$$LCy'' + \left(R_1 C + \frac{L}{R_2}\right) y' + \left(1 + \frac{R_1}{R_2}\right) y = 0$$

or in standard form:

$$\boxed{y'' + \left(\frac{L + R_1 R_2 C}{R_2 L C}\right) y' + \left(\frac{R_1 + R_2}{R_2 L C}\right) y = 0}$$

2 $\quad y'' + 6y' + 34y = 5x \qquad y(0^-) = 2$

$y'(0^-) = 0$

a) Find zero-input response.

$Q(D) = D^2 + 6D + 34$ has roots $\dfrac{-6 \pm \sqrt{36 - 4(34)}}{2}$

$-3 \pm j5$

$y_0(t) = e^{-3t}(C_1 \cos(5t) + C_2 \sin(5t))$

$y_0'(t) = e^{-3t}(-5C_1 \sin(5t) + 5C_2 \cos(5t))$

$+ -3e^{-3t}(C_1 \cos(5t) + C_2 \sin(5t))$

$y_0(0^-) = \underline{C_1 = 2}$

$y_0'(0^-) = 5C_2 - 3C_1 = 0 \qquad C_2 = \dfrac{3C_1}{5} = \underline{\dfrac{6}{5}}$

$$\boxed{y_0(t)u(t) = e^{-3t}\left(2\cos(5t) + \frac{6}{5}\sin(5t)\right)u(t)}$$

b) Find impulse response. $Q(D)$ roots from a) $-3 \pm j5$

$P(D) = \underset{b_0}{0}D^2 + \underset{b_1}{0}D + \underset{b_2}{5}$

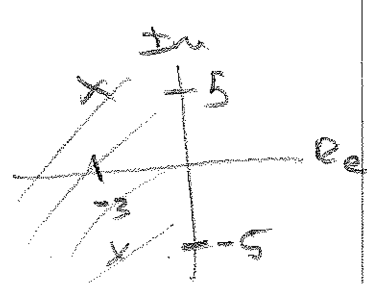
$y_n(t) = e^{-3t}(C_1 \cos(5t) + C_2 \sin(5t))$

From a) and $y_n(0^+) = 0 \quad y_n'(0^+) = 1$

$C_1 = 0 \qquad 5C_2 = 1$ or $C_2 = \dfrac{1}{5}$

$y_n(t) = \dfrac{1}{5} e^{-3t} \sin(5t)$

$h(t) = \underset{\substack{\Downarrow \\ 0}}{b_0} \delta(t) + \underset{\substack{\Downarrow \\ 5}}{[P(D) y_n]} u(t)$



$$\boxed{h(t) = e^{-3t}\sin(5t)\,u(t)}$$

c) Determine Lyapunov stability: 2 roots are in LHP so $\underline{\underline{\text{Stable}}}$

(3) $$h(t) = 2\left(e^{-t} + e^{-3t}\right)u(t)$$

$$x(t) = 2e^{-3t+6}\,u(t-2)$$

Find $y_{zs}(t) = (h * x)(t)$

Lets first do some factoring of h

$$h(t) = \underbrace{2e^{-t}u(t)}_{h_1} + \underbrace{2e^{-3t}u(t)}_{h_2}$$

Lets rewrite $x(t)$ as

$$x(t) = 2x_1(t-2) \qquad -3t+6 = a(t-2) \text{ when } a = -3$$

$$x_1(t) = e^{-3t}u(t)$$

Then $y_{zs}(t-2) = (h_1 * x_1)(t) + (h_2 * x_1)(t)$

using TABLE 2.1 row 4 $$(h_1 * x_1) = \left[2e^{-t}u(t)\right] * \left[e^{-3t}u(t)\right]$$
$$= 2\,\frac{\left(e^{-t} - e^{-3t}\right)}{2}\,u(t)$$
$$= \left(e^{-t} - e^{-3t}\right)u(t)$$

using TABLE 2.1 row 5 $$(h_2 * x_1) = \left[2e^{-3t}u(t)\right] * \left[e^{-3t}u(t)\right]$$
$$= 2te^{-3t}u(t)$$

Thus $$y_{zs}(t) = 2\left[e^{-t} + (2t-1)e^{-3t}\right]u(t)\Big|_{t \to t-2}$$

$$y_{zs}(t) = 2\left[e^{-t+2} + (2(t-2)-1)e^{-3t+6}\right]u(t-2)$$
$$= 2\left[e^{-t+2} + (2t-5)e^{-3t+6}\right]u(t-2)$$
$$= \left[2e^{-t+2} + 4te^{-3t+6} - 10e^{-3t+6}\right]u(t-2)$$

4. $h(t) = (2e^{-t} + 2e^{-3t})u(t)$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{0}^{\infty} |\underbrace{2e^{-t} + 2e^{-3t}}_{\text{always positive}}|\, dt$$

$$= \int_0^{\infty} 2e^{-t}\, dt + \int_0^{\infty} 2e^{-3t}\, dt$$

$$= -2e^{-t}\Big|_0^{\infty} + -\frac{2}{3}e^{-3t}\Big|_0^{\infty}$$

$$= +2 + \frac{2}{3} = \frac{8}{3} < \infty \quad \therefore \underline{\text{BIBO stable}}.$$

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5. $h_1(t) = e^{-t}u(t) \qquad h_2(t) = te^{-2t}u(t)$

$\longrightarrow \boxed{h_1} \longrightarrow \boxed{h_2} \longrightarrow$

$\longrightarrow \boxed{h_1 * h_2} \longrightarrow$

$$(h_1 * h_2)(t) = \frac{e^{-t} - e^{-2t} + (-2+1)te^{-2t}}{(-2+1)^2}\, u(t)$$

Row 9 of table 2.1

$\lambda_1 = -2$

$\lambda_2 = -1$

$$= \frac{e^{-t} - e^{-2t} - te^{-2t}}{1}\, u(t)$$

$$= \underline{\left[e^{-t} - (1+t)e^{-2t}\right]u(t)}$$