- interconnected systems
- response to complex exponential
- total response
- examples

As systems get larger it becomes easier to reason about them by dividing them into blocks.

Let's look at simple cases for now:

**Parallel blocks**

\[ x(t) \xrightarrow{S_1} y_1(t) \quad y_1(t) = h_1(t) \]
\[ x(t) \xrightarrow{S_2} y_2(t) \quad y_2(t) = h_2(t) \]
\[ h(t) = y_1(t) + y_2(t) \]

**Series blocks**

\[ x(t) \xrightarrow{S_1} y_1(t) \quad y_1(t) = h_1(t) \]
\[ y_1(t) \xrightarrow{S_2} y_2(t) \quad y_2(t) = h_2(t) \]
\[ h(t) = y_1(t) \ast h_2(t) \quad \text{convolution.} \]

**Integrator**

\[ x(t) \xrightarrow{\int} y(t) = \int_{-\infty}^{t} x(\tau) \, d\tau \]

**Derivative**

\[ x(t) \xrightarrow{\frac{d}{dt}} y(t) = \frac{dx}{dt} \]
- When describing system using blocks we ignore loading.

Ex. Suppose \( s_1 \) was

\[
\begin{align*}
R_1 & = \frac{V}{I} \\
R_2 & = L \\
h_1 & = e^{-\frac{t}{R_1 C}} \\
h_2 & = e^{-2t}
\end{align*}
\]

Thus \( s_2 \) was

\[
\begin{align*}
y_1' + \frac{1}{RC_1} y_1 & = \frac{1}{C_1} x \\
y_2' + \frac{R_2}{L} y_2 & = \frac{R_2}{L} x
\end{align*}
\]

\[
h_1 = \frac{1}{R_1 C} e^{-\frac{t}{R_1 C}} \\
h_2 = \frac{R_2}{L} e^{-\frac{R_2 t}{L}}
\]

Using blocks

\[
\begin{align*}
x(t) & \rightarrow [s_1] \rightarrow [s_2] \rightarrow y(t) \\
x(t) & \rightarrow [h_1 \ast h_2] \rightarrow y(t)
\end{align*}
\]

\[
h(t) = (e^{-t} - e^{-2t}) u(t)
\]

Suppose we nicely connected those two systems.

\[
\begin{align*}
R_1 & \\
R_2 & \\
h(t) & = h_1 \ast h_2
\end{align*}
\]

This gives the second ODE

\[
y'' + 3y' + 2y = 2x
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This has complex roots, so we know the impulse response will be sinusoidal!

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Response to complex exponential $e^{st}$ (non-zero $t$) 

$$x(t) \rightarrow [h(t)] \rightarrow y(t) = h \ast x$$

$$y(t) = (h \ast x)(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

$$= \int_{-\infty}^{0} h(\tau) e^{-s\tau} d\tau + \int_{0}^{\infty} h(\tau) e^{-s\tau} d\tau$$

$$= e^{st} \int_{-\infty}^{0} h(\tau) e^{-s\tau} d\tau = H(s) e^{st}$$

Transfer Function, $s$-function

$$h(t)$$

In terms of the DE,

$$[Q(D)] y = [P(0)] x$$

$$\downarrow s^t \quad \downarrow s^t$$

$$H(s) e^{st} \quad e$$

$$D^n [H(s) e^{st}] = H(s) s^n e^{st}$$

$$[Q(D)] H(s) e^{st} = H(s) Q(s) e^{st} = P(s) e^{st}$$

$$H(s) = \frac{P(s)}{Q(s)}$$

We will return to this section in Chapter 4.
To summarize then, the total response

\[ y(t) = y_0(t) + y_{2s}(t) = y_0(t) + h(t) * x(t) \]

*Zero-input response*  
*Zero-state response*  
*Impulse response*

If \( \lambda_k \) are the roots of \( Q(\lambda) \) then,

\[ y(t) = \sum_{k=1}^{N} C_k e^{\lambda_k t} + h(t) * x(t) \]

Example:

\[ h(t) = R = 2, \quad c = \frac{1}{2}, \quad x(t) = u(t) \]

\[ y' + y = x \]

\[ y_0(t) = ce^{-t}, \quad y_0(0) = 1 \]

\[ y_r(t) = e^{-t} \]

\[ h(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \]

\[ y(t) = \int_{-\infty}^{0} x(\tau) h(t-\tau) d\tau \]

- For \( t < 0 \)

- For \( t > 0 \)

Together,

\[ y_{2s}(t) = (1 - e^{-t}) u(t) \]

*Zero-state response*
Suppose there was an initial charge on the capacitor $v \frac{1}{2} \text{V}$, $y(0^-) = \frac{1}{2}$, then $y_0(t) = \frac{1}{2} e^{-t}$.

The total response is then:

$$
y(t) = \frac{1}{2} e^{-t} u(t) + (1 - e^{-t}) u(t)
$$

$$
= (1 - \frac{1}{2} e^{-t}) u(t)
$$