Classifying Systems.

- Recall a system is a Transformation between functions,

$$x(t) \rightarrow \boxed{T} \rightarrow y(t)$$

- A system is <u>linear</u> if the principle of superposition holds.

if $\quad x_1(t) \rightarrow \boxed{T} \rightarrow y_1(t)$

and $\quad x_2(t) \rightarrow \boxed{T} \rightarrow y_2(t)$

then

$$a x_1(t) + b x_2(t) \longrightarrow \boxed{T} \rightarrow a y_1(t) + b y_2(t)$$

constants a, b in $\mathbb{R}$

otherwise it is non-linear.

Example: Consider a system described by the 1st order DE

$$y' + ay = x \qquad \text{for } a \in \mathbb{R}$$

The solution is $\quad y(t) = e^{-at}\int e^{at} x(t)\, dt$

Let $\quad y_1(t) = e^{-at}\int e^{at} x_1(t)\, dt$

$$y_2(t) = e^{-at}\int e^{at} x_2(x)\, dt$$

Let $\quad x(t) = b x_1(t) + c x_2(t)$

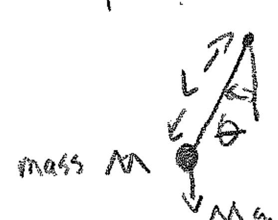
$$y(t) = e^{-at}\int e^{at}(b x_1 + c x_2)\, dt$$

$$= b e^{-at}\int e^{at} x_1(t)\, dt + c e^{-at}\int e^{at} x_2(t)\, dt$$

$$= b y_1(t) + c y_2(t)$$

$\therefore$ The system is linear.

Example: An example of a non-linear system is the pendulum, treating the initial angle as the input.



$$\ddot{\theta} + \frac{g}{L}\underbrace{\sin(\theta)} = 0$$

$$\theta(0) = \theta_0$$

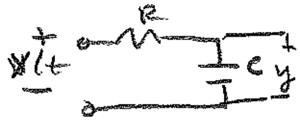
— A system is time-invariant if the response does not depend on <u>when</u> the input is applied.

if $x(t) \rightarrow \boxed{T} \rightarrow y(t)$

then $x(t-\tau) \rightarrow \boxed{T} \rightarrow y(t-\tau)$

- Any constant coefficient differential Equation is time-invariant.

Example: Consider the series RC - circuit



$\Rightarrow \quad y' + \frac{1}{RC} y = \frac{1}{RC} x$

this is a <u>constant</u> coefficient DE as long as the values of R & C do not change.

Note this is an approximation, real circuit elements do change over time and as a function of environment, hopefully only a little and slowly.

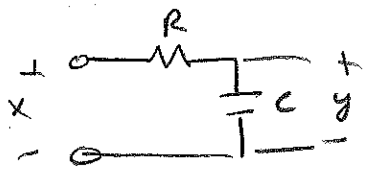
— A system is memoryless if the output only depends on the current input. i.e. $y(t) = F(x(t))$

otherwise the system has memory and is dynamic.

- Any system with a (non-trivial) differential equation description is dynamic.

Example:

memoryless



dynamic

— A system is <u>causal</u> if the response depends only on the current and past values of the input.

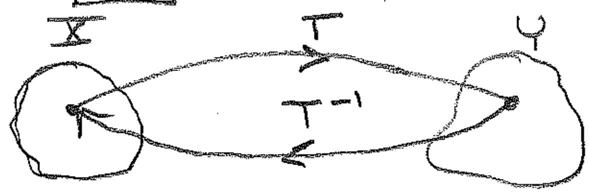
i.e. the system cannot predict the future.

$$y(t_0) = F(x(t)) \text{ for } t \le t_0 \text{ only.}$$

- All physical systems are causal.

— A system is invertible if you can obtain the input from $y(t)$ the output.

$$x(t) \rightarrow \boxed{T} \xrightarrow{y(t)} \boxed{T^{-1}} \rightarrow x(t)$$



• This concept is useful in e.g. communication.

$$x(t) \rightarrow \boxed{\text{Encode}} \rightarrow \text{Channel} \rightarrow \boxed{\text{Decode}} \rightarrow x(t)$$

inverse systems

AND system identification.

$$x(t) \rightarrow \boxed{?} \rightarrow y(t) \rightarrow \boxed{T^{-1}} \rightarrow x(t)$$

to find T ⟵ find T^{-1}

Note, in general this is an ill posed problem.

— Stability: There are two definitions of stability internal and external stability.

They are usually equivalent (but not always).

For now we will describe external stability also called Bounded Input, Bounded Output or BIBO.

Recall a signal is Bounded if $x(t) < \infty \ \forall t$

A system is BIBO stable if

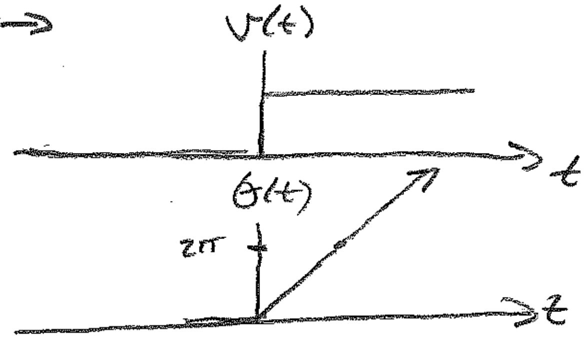
$$\underset{\text{bounded}}{x(t)} \rightarrow \boxed{} \rightarrow \underset{\text{is bounded.}}{y(t)}$$

* Stability is important because any practically useful system must be stable.

- Note, stability can depend on the space of the output / interpretation of the signal.

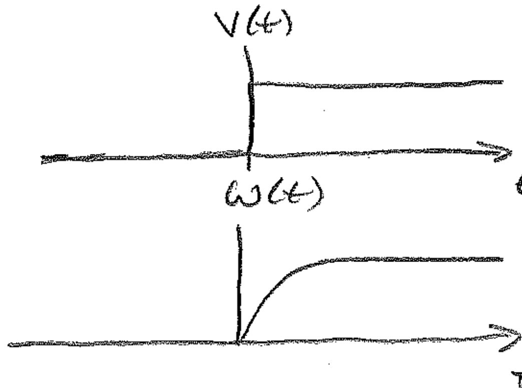
Example: Consider a DC-motor.

if we consider the angle of the shaft as the output.

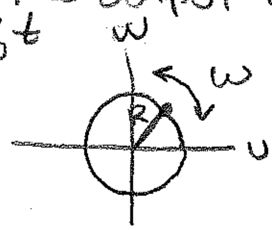


It appears to be an unstable system.

- However, if we consider the rotational velocity as the output



- we could also treat the output as the cartesian coordinate of the shaft



$u = R\cos\theta$

$w = R\sin\theta$

---

* In this course we will study linear, time-invariant systems (LTI) usually dynamic & causal.

- might or might not be stable, which will become a deciding factor in what techniques we use.