★ Introduction to Systems.
- System Definition
- System Modeling
- Examples

- A system is a transformation between one or more signals, a rule that maps functions to functions

$$x(t) \longrightarrow \boxed{T} \longrightarrow y(t)$$

- single input - single output (SISO)

$$x(t) \longrightarrow \boxed{T} \begin{matrix} \rightarrow y_0(t) \\ \rightarrow y_1(t) \\ \vdots \\ \rightarrow y_n(t) \end{matrix} \Bigg\} \text{ internal signals.}$$

- single input - multiple output (SIMO)

- The most general case is

$$\begin{matrix} x_0 \rightarrow \\ x_1 \rightarrow \\ \vdots \\ x_n \rightarrow \end{matrix} \boxed{T} \begin{matrix} \rightarrow y_0 \\ \rightarrow y_1 \\ \vdots \\ \rightarrow y_n \end{matrix}$$

multiple input multiple output (MIMO)

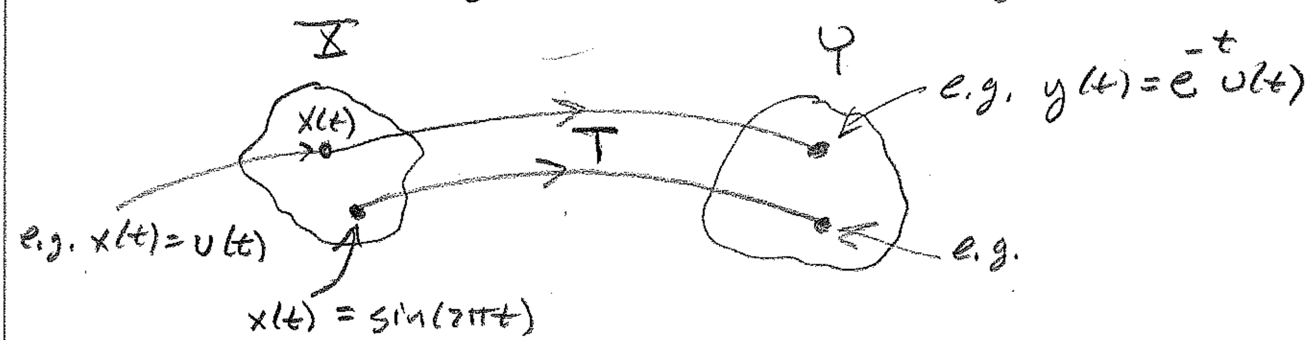
- Recall signals are functions from independent variables to a range of values.

We can view system similarly

$$t \rightarrow \boxed{T} \rightarrow f(t) \qquad x(t) \rightarrow \boxed{T} \rightarrow y(t)$$

Let $\mathbb{X}$ be the set of all input signals
$\mathbb{Y}$ be the set of all possible output signal

Then a system T is a mapping from $\mathbb{X}$ to $\mathbb{Y}$



- When $\mathbb{X}$ & $\mathbb{Y}$ are the set of CT signals we call it a continuous-time system.

- When $\mathbb{X}$ & $\mathbb{Y}$ are the set of DT signals we call it a discrete-time system

- when $\mathbb{X}$ & $\mathbb{Y}$ are different sets, e.g. $\mathbb{X}$ DT, $\mathbb{Y}$ CT, we call it a <u>hybrid</u> or mixed system.
- when there is no input, the system is autonomous.

* Examples: (what are inputs, outputs?)

- a circuit consisting of R, L, C, op-Amps?
- a music program like audacity or GarageBAND?
- an mp3 player? (go over in detail)
- breaking system in a car?
- animals (including humans)?
- an interstate highway?

This list goes on-and-on, almost any thing of interest that changes with time can be viewed as a system.

- We can represent systems mulitple ways
  - mathematically
  - graphically

  and we can convert back and forth.

- In general, for systems the most common (and most widely applicable) mathematical representation is the differential equation.

differential equation for CT systems ← $\dot{\bar{u}} = f(\bar{x}, \bar{u}, t)$

$\bar{y} = g(\bar{x}, \bar{u}, t)$ ← output equation

$\bar{x}$ inputs

$\bar{u}$ internal states

$\bar{y}$ outputs

- The rationale for this choice is DE's are equations whose solution is a function.

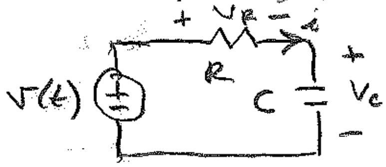
- Different kinds of systems correspond to different kinds of DE's, some with <u>simpler</u> mathmatical representations.

  For us, the class of Linear, time-invariant systems,

* Let's look at some examples. To model the system we

Modeling { 1. identify input output
2. derive a differential equation representation

- Consider the series R-C circuit.



1. input is voltage signal, output could be
   - loop current
   - resistor voltage
   - capacitor votage.

2. to derive the DE we use the laws of circuits.

KCL: $\frac{v(t) - V_c(t)}{R} = C V_c'(t)$

rearranging: $V_c'(t) = -\frac{1}{RC} V_c(t) + \frac{1}{RC} v(t)$

$\Downarrow$

$\dot{u} = f(x, u)$ $\quad x = v(t)$, $u = V_c(t)$

$f = -au + bx$

The output equation depends on problem.

- Interested in capacitor voltage.

$y = V_c \Rightarrow y = g(x, u) = u$

- Interested in Resistor voltage.

$y = V - V_c \Rightarrow y = g(x, u) = x - u$

- Interested in loop current.

$y = \frac{V - V_c}{R} \Rightarrow y = g(x, u) = a(x - u)$

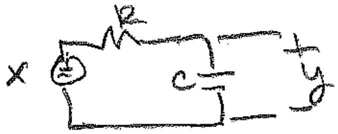
- So we can view this as a CT SISO system

Graphically | Mathematically

Externe

TBD ? (externe)

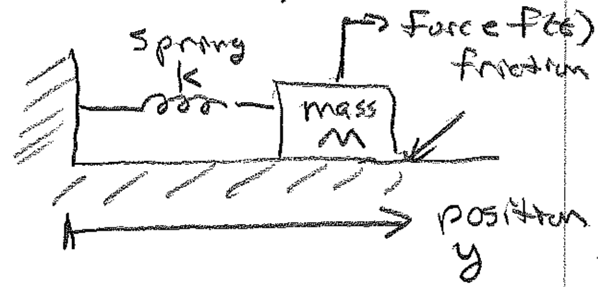
Interne



$\dot{u} = -au + bx$
$y = u$ $\quad a = b = \frac{1}{RC}$ (interne)

- It does not matter where the DE comes from.

Example: mechanical system



y = position $\quad M$ = mass

y' = velocity $\quad k$ = spring constant

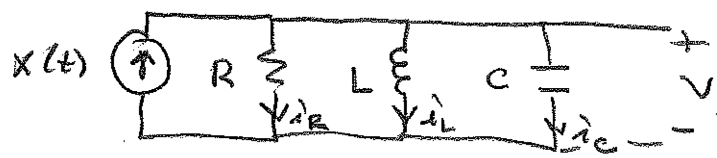
y'' = acceleration $\quad B$ = coeff of friction.

Newton's Laws: Force $= My'' = -ky - By' + x$

or $\boxed{y'' + \frac{B}{M}y' + \frac{k}{M}y = \frac{1}{M}f}$

Free Body Diagram

- Example: Parallel RLC with a current source.



Physical Laws

$i_C = C v_C'$

$V_L = L i_L'$

KCL sum current is zero

KCL: $x = \frac{V}{R} + i_L + CV'$

$V = Li' \qquad V' = Li''$

$$i_L'' + \frac{1}{RC} i_L' + \frac{1}{LC} i_L = \frac{1}{LC} x(t)$$

Treating i_L as the output of interest, y.

$$y'' + \frac{1}{RC} y' + \frac{1}{LC} y = \frac{1}{LC} x$$

- Compare this to the previous example.

$R = \frac{1}{B} \qquad L = \frac{1}{k} \qquad C = M \qquad$ they are identical.

To translate between mechanical + electrical systems

voltages	$\longrightarrow$	velocities	or	force
currents	$\longrightarrow$	forces		velocity
resistance	$\longrightarrow$	$\frac{1}{\text{friction}}$		friction
Capacitance	$\longrightarrow$	mass		compliance
Inductance	$\longrightarrow$	Compliance $\frac{1}{k}$		mass.