**Introduction to Systems.**
- System Definition
- System Modeling
- Examples

A system is a transformation between one or more signals, a rule that maps functions to functions.

\[ X(t) \rightarrow T \rightarrow Y(t) \]

- **Single Input - Single Output (SISO)**
  \[ X(t) \rightarrow T \rightarrow Y(t) \]

- **Single Input - Multiple Output (SIMO)**
  \[ X(t) \rightarrow T \rightarrow Y(t) \]

- **Multiple Input - Multiple Output (MIMO)**

Recall signals are functions from independent variables to a range of values.

We can view system similarly.

\[ t \rightarrow T \rightarrow f(t) \]

\[ x(t) \rightarrow T \rightarrow y(t) \]

Let \( X \) be the set of all input signals.

\( Y \) be the set of all possible output signals.

Then a system \( T \) is a mapping from \( X \) to \( Y \).

\[ X \rightarrow T \rightarrow Y \]

**Examples:**

- \( x(t) = e^{-t} \)

- \( x(t) = \sin(\pi t) \)

- When \( X \) and \( Y \) are the set of \( eT \) signals we call it a continuous-time system.

- When \( X \) and \( Y \) are the set of \( DT \) signals we call it a discrete-time system.
- When \( x \) and \( y \) are different sets, e.g., \( \mathbb{R} \) or \( \mathbb{C} \), we call it a hybrid or mixed system. If \( x \) or \( y \) is a system, then it is autonomous. Examples: What are inputs, outputs?
  - a circuit consisting of R, L, C, op-Amps?
  - a music program like Audacity or GarageBand?
  - an MP3 player? (go over in detail)
  - breaking system in a car?
  - animals (including humans)?
  - an interstate highway?

This list goes on and on, almost anything of interest that changes with time can be viewed as a system.

- We can represent systems multiple ways
  - mathematically
  - graphically
  - and we can convert back and forth.

- In general, for systems the most common (and most widely applicable) mathematical representation is the differential equation.

\[
\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, t) \\
\mathbf{y} = g(\mathbf{x}, \mathbf{u}, t)
\]

- The rationale for this choice is DEs are equations whose solution is a function.

- Different kinds of systems correspond to different kinds of DEs, some with simpler mathematical representations.

For us, the class of linear, time-invariant systems,
Let's look at some examples. To model the system, we identify input output.

1. Identify input output
   - Consider two series R-C circuit.
   - Input is voltage signal, output could be
     - Loop current
     - Resistor voltage
     - Capacitor voltage.

2. To derive the DE, we use the laws of circuits.
   - KCL: $\frac{v(t) - v_c(t)}{R} = c \frac{v_c'(t)}{R}$
   - Rearrange: $v_c'(t) = -\frac{1}{RC} v_c(t) + \frac{1}{RC} v(t)$
   - $\dot{y} = f(x, u)$, $x = v(t)$, $u = v_c(t)$
   - $f = -au + bx$

The output equation depends on the problem.

- Interested in capacitor voltage.
  - $y = v_c \Rightarrow u = g(x, u) = u$

- Interested in Resistor voltage.
  - $y = v - v_c \Rightarrow y = g(x, u) = x - u$

- Interested in loop current.
  - $y = \frac{v - v_c}{R} \Rightarrow y = g(x, u) = a(x - u)$

- So we can view this as a CT SISO system

**Graphically**

**Mathematically**

Extends $x(t) \rightarrow \square \rightarrow y(t)$

$\dot{u} = -au + bx$ (internal)

$y = u \quad a = b = \frac{1}{RC}$ (external)
It does not matter where the DE comes from.

**Example: Mechanical System**

\[ y = \text{position} \quad M = \text{mass} \]
\[ y' = \text{velocity} \quad k = \text{spring constant} \]
\[ y'' = \text{acceleration} \quad B = \text{coefficient of friction} \]

**Newton's Laws:**

\[
y'' + \frac{B}{M} y' + \frac{k}{M} y = \frac{1}{M} f
\]

Free Body Diagram

**Example: Parallel RLC with a Current Source.**

**Physical Laws**

\[ \dot{I}_C = CV_i \]
\[ V_L = L \dot{I}_L \]

**KCL Sum Current is Zero**

\[
R \frac{d^2 I}{dt^2} + L \frac{d I}{dt} + \frac{1}{C} I = \frac{1}{C} x(t)
\]

Treating \( I \) as the output of interest, \( y \).

\[
y'' + \frac{1}{RC} y' + \frac{1}{LC} y = \frac{1}{LC} x
\]

*Compare this to the previous example.*

\[
R = \frac{1}{B} \quad L = \frac{1}{k} \quad C = M \quad \text{they are identical.}
\]

To translate between mechanical and electrical systems:

- **Voltages** \( \rightarrow \) **Velocities** or **Force**
- **Currents** \( \rightarrow \) **Forces**
- **Resistance** \( \rightarrow \) **Friction**
- **Capacitance** \( \rightarrow \) **Mass**
- **Inductance** \( \rightarrow \) **Compliance** or \( \frac{1}{k} \)