- Signal Classifications
- the complex exponential.

\# Signal Classification

Recall from Lecture 2 that signals are modeled as functions $f: A \to B$ for sets A (domain) and B (co-domain)

The range of a function is the actual set of values it can take on.

e.g. $f: \mathbb{R} \to \mathbb{R}$ $\quad f(t) = e^t$

$\forall t$ this is positive so the range is $\mathbb{R}^+$

- If a signal has a range that is Bounded to some subset, then we say the signal is <u>Bounded</u>.

e.g. e^t is unbounded $\quad \sin(t)$ is bounded

This distinction will be important when we talk about Stability of systems.

- if a signal is zero outside some finite range it is <u>time-limited</u>.
- If the domain of the function is $\mathbb{R}$ we call this a <u>continuous-time signal</u>. $f(t)$
- If the domain of the function is $\mathbb{Z}$, the integers, we call this a discrete-time signal. $f[n]$
- If the range is a subset of $\mathbb{R}$ we call this an <u>analog</u> signal.
- If the range is a subset of $\mathbb{Z}$ we call this a <u>digital</u> signal.

- Signals for which $f(t+T) = f(t) \quad \forall t, T \in \mathbb{R}$

or $f[n-M] = f[n] \quad \forall n, M \in \mathbb{Z}$

we say the signal is <u>periodic</u>. Otherwise it is aperiodic / non periodic.

Let $T = nT_0$ for some $T_0 \in \mathbb{R}$ and $n \in \mathbb{Z}^+$

then T_0 is the <u>fundamental Period</u>

Example: $x(t) = \sin\left(\frac{2\pi}{T_0} t\right)$

$f_0 = \frac{1}{T_0}$ is the frequency in Hz $\quad \omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$ is frequency radian/sec.

- Signals for which the Energy is finite are Energy signals
- Signals with finite, non-zero power are called Power Signals.

N • Signals may be Energy, Power or <u>Neither</u>

- Power signals need not be periodic, but periodic signals who are bounded are power signals.

$\boxed{\text{DEMO}}$

- All practical signals are energy signals, however for some kinds of analysis it is convienient (and accurate) to model them as power signals.

- Signals whose function is deterministic are said to be <u>deterministic</u>.
- Signals described by random / stochastic variables are random / stochastic signals.
  - The course only treats deterministic signals, however stochastic signals play an important role in more advanced signals & systems. See e.g. ECE 5604

Lets try some examples: $t \in \mathbb{R}$ $n \in \mathbb{Z}$

make slide

<u>function</u>	<u>Bounded?</u>	<u>CT/DT?</u>	<u>A/D?</u>	<u>periodic?</u>	<u>E/P?</u>	<u>D/S?</u>
$e^{-t}u(t)$						
$t\,u(t)$						
e^{j3t}						
$\sin(3n)$						
e^{t^2}						
$\frac{1}{t}u(t)$						
$e^{t}u(-t)$						
$\frac{1}{n}+\frac{1}{n^2}$						

☆ The most important signal in this course is the complex exponential

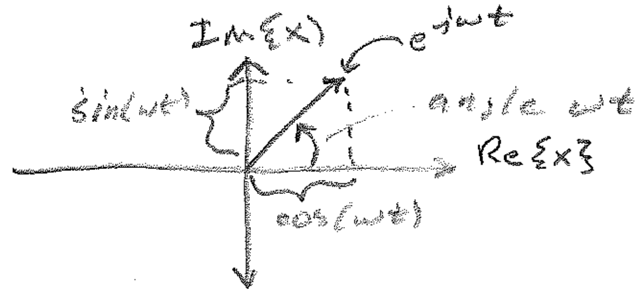
$$x(t) = e^{st} \quad \text{where } e \text{ is Euler's number}$$

$s \in \mathbb{C}$ $\qquad$ $e \approx 2.72$

$s = \sigma + j\omega \quad a, \omega \in \mathbb{R}$ $\qquad$ $e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \dots$

- we can split e^{st} into $e^{\sigma t} e^{j\omega t}$
- and use Euler's identity $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$
- when $\sigma = 0$ we call this a rotating unit phasor

this vector rotates as t increases.

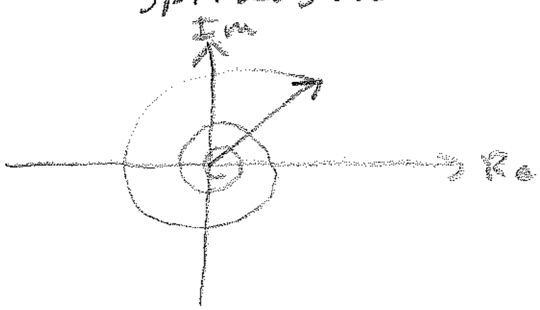


$\boxed{\text{DEMO}}$

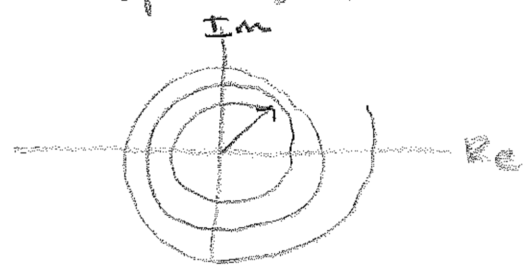
See problem in PS0

- when $\sigma \neq 0$ the magnitude of the vector also changes with time.

$\sigma < 0$
"spirals in"



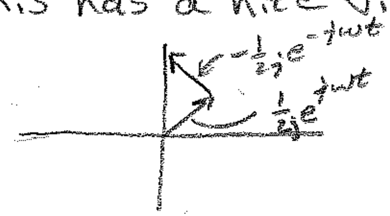
$\sigma > 0$
"spirals out"

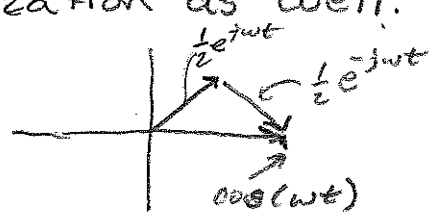


- Another pair of relations that is instructive is

$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \qquad \cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

This has a nice visualization as well. $\boxed{\text{DEMO}}$





* Why is e^{st} so important?

Recall from linear algebra, given a vector $x \in \mathbb{R}^n$ and a square matrix A $(n \times n)$

$T(\bar{x}) = Ax$ defines a linear transformation.

There exist special vectors v_i $i = 1..n$ such that

$A v_i = \lambda_i v_i$ the eigenvectors with the associated eigenvalues, λ_i

This is important because we can use the v_i as a convenient basis.

$$x = \sum_{i=1}^{n} a_i v_i$$

then Ax becomes $\sum_{i=1}^{n} \underbrace{\lambda_i a_i}_{\text{multiplication}} v_i$

Similarly it will turn out for the systems of interest to use in this course

Let $x(t)$ be a signal

$T\{x(t)\}$ is the transformation (Linear Time Inv)

the signal $v(t) = e^{-st}$ $s = \sigma + j\omega$ is the <u>eigenfunction</u> for the system T.

if we can write $x(t)$ in this basis it simplifies the analysis.

$$H(s) = \int_0^\infty h(t) e^{-st} dt$$

eigenvalues —— Eigenfunction.

$$X(s) = \int_0^\infty x(t) e^{-st} dt$$

basis coordinates —— Eigenfunction.

$T\{x(t)\}$ becomes $H(s)X(s)$

this is the Laplace Transform.