– we model signals (mathematically represent them)

by combining elementary functions.

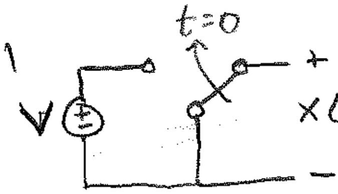
polynomials, transcendental functions, piece-wise

t, t^2 $\underbrace{e^{\lambda t}}_{\text{eigen function}}, \underbrace{\cos, \sin}_{\text{basis.}}$ $\begin{cases} f_1(t) & t<0 \\ f_2(t) & t \geq 0 \end{cases}$

and a special function we will talk about next week, the impulse function $\delta(t)$

Examples:

- Switch Model



$$x(t) = \begin{cases} 0 & t<0 \\ V & t \geq 0 \end{cases}$$

we use this model so much we give it its own name and symbol

unit step $u(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$

- Pure Audio tone at "middle C"

$\boxed{\text{DEMO}}$

$$x(t) = \sin(2\pi(261.6)t)$$

- The chord "G" (G, B, D notes)

$$x(t) \approx \sin(2\pi(392)t) + \sin(2\pi(494)t) + \sin(2\pi(293)t)$$

– So we can use addition to build up signals to approximate real signals of interest. (design)

<u>or</u>

(Analyze) real signals to see what signals make them up.

– we can also apply some basic transformations to the primitive signals

* Introduction to Signals.
  - Signal Energy
  - Signal Power
  - Transformations

Input Signal $\longrightarrow$ $\boxed{\text{System}}$ $\longrightarrow$ Output Signal

* Signals are modeled as functions.

A function f is a mapping from elements of the domain to those in the co-domain.

$$f: \text{Domain} \longrightarrow \text{Co-Domain}$$

Examples:

$f: \mathbb{R} \to \mathbb{R} \Rightarrow$ analog, continuous-time signal

e.g. a voltage at time t, $v(t)$

$f: \mathbb{R} \to \mathbb{Z} \Rightarrow$ digital, continuous-time signal

e.g. the output of a GPIO port on a microcontroller

$f: \mathbb{Z} \to \mathbb{R} \Rightarrow$ analog, discrete-time signal.

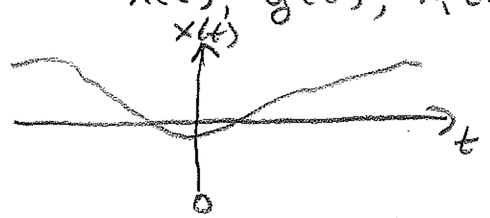
e.g. the temperature every day at noon.

$f: \mathbb{Z} \to \mathbb{Z} \Rightarrow$ digital, discrete-time signal

e.g. signal on a computer bus $\xrightarrow{\;/\;}$ n-bits.

- The focus of this course is on continuous-time signals.

$x(t)$, $y(t)$, $x_1(t)$, $x_2(t)$ etc.



↑ time reference (often 0)

* We can distinguish between the real, physical signal and it's mathematical representation.

E.g. $x(t)$ = pressure change of air (sound)

↑

How do we model such signals?

* Common Signal transformations

- Magnitude Scaling $x_2(t) = A\,x_1(t)$ $A \in \mathbb{R}$
- derivatives $x_2(t) = D\,x_1(t)$
- integrate $x_2(t) = \int_{-\infty}^{t} x_1(\tau)\,d\tau$
- sums $y(t) = \sum_{i=1}^{N} x_i(t)$

an important example of this we will see is the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \cos(b_k t + c_k)$$

- multiplication (Modulation)

$$y(t) = x_1(t)\,x_2(t)$$

e.g. AM Radio $y(t) = x(t)\sin(\omega_0 t)$

$x(t)$ is audio signal
ω_0 is the carrier frequency

- time shift $x_2(t) = x_1(t - \tau)$ $\tau > 0$ delay, $\tau < 0$ advance

$\boxed{\text{DEMO}}$

- time scaling $x_2(t) = x_1\left(\frac{t}{\tau}\right)$

for $\tau > 0$ increasing τ expands in time, slows the signal down

for $\tau < 0$ time reverses and decreasing τ expands in time.

e.g. $\tau = -1$ is pure time reversal.

$$x(t) = e^{t} \qquad x(-t) = e^{-t}$$

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* used in combination these transformations give us a powerful way to model signals in real applications.

* A useful (for reasons we will see later) measure of a signal is it's Energy, the squared norm of signal.

- The energy of a CT signal is

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

- The power of a CT signal is the average energy over an interval T.

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \quad \text{or} \quad P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

- Note: the units of Energy and Power depend on the physical interpretation of the signal.

  E.g. <u>if</u> the signal is a current in a circuit, then E_x is in Joules, P_x is Watts.

Example: Given $x(t) = \begin{cases} 0 & t < 0 \\ e^{at}\cos(t) & t \geq 0 \end{cases}$

What is the energy & Power E_x, P_x?

a) Plot for various values of a, $a<0$, $a=0$, $a>0$

b) Plot $|x(t)|^2$ for various values of a, discuss what the integral means e.g. $a<0$ gives $E_x < \infty$

c) Compute the integrals for E_x, P_x.

$\boxed{\text{DEMO}}$