ECE 5984: Introduction to Machine Learning

Topics:

- SVM
  - Multi-class SVMs
- Neural Networks
  - Multi-layer Perceptron

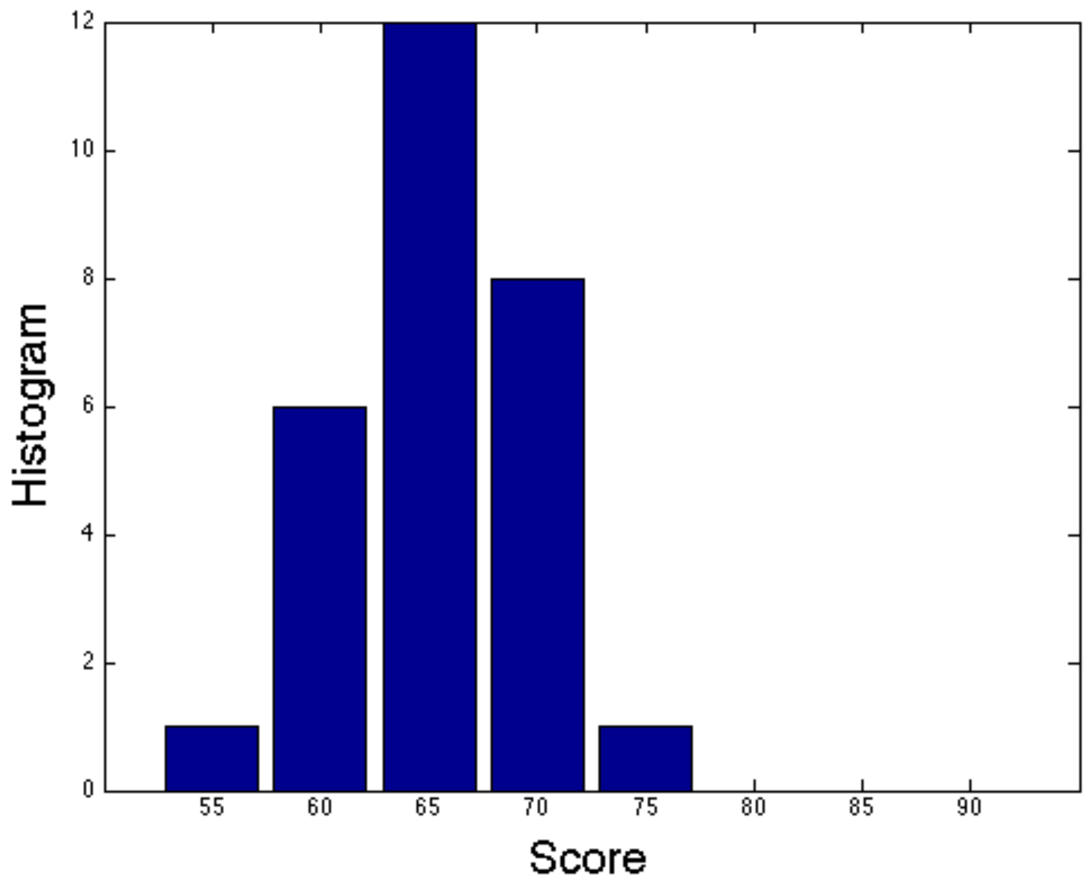
Readings: Barber 17.5, Murphy 16.5

Dhruv Batra

Virginia Tech

HW2 Graded

- Mean 66/61 = 108%
  - Min: 47
  - Max: 75



Administrativia

- HW3
  - Due: in 2 weeks
  - You will implement primal & dual SVMs
  - Kaggle competition: Higgs Boson Signal vs Background classification
  - https://inclass.kaggle.com/c/2015-Spring-vt-ece-machine-learning-hw3
  - https://www.kaggle.com/c/higgs-boson

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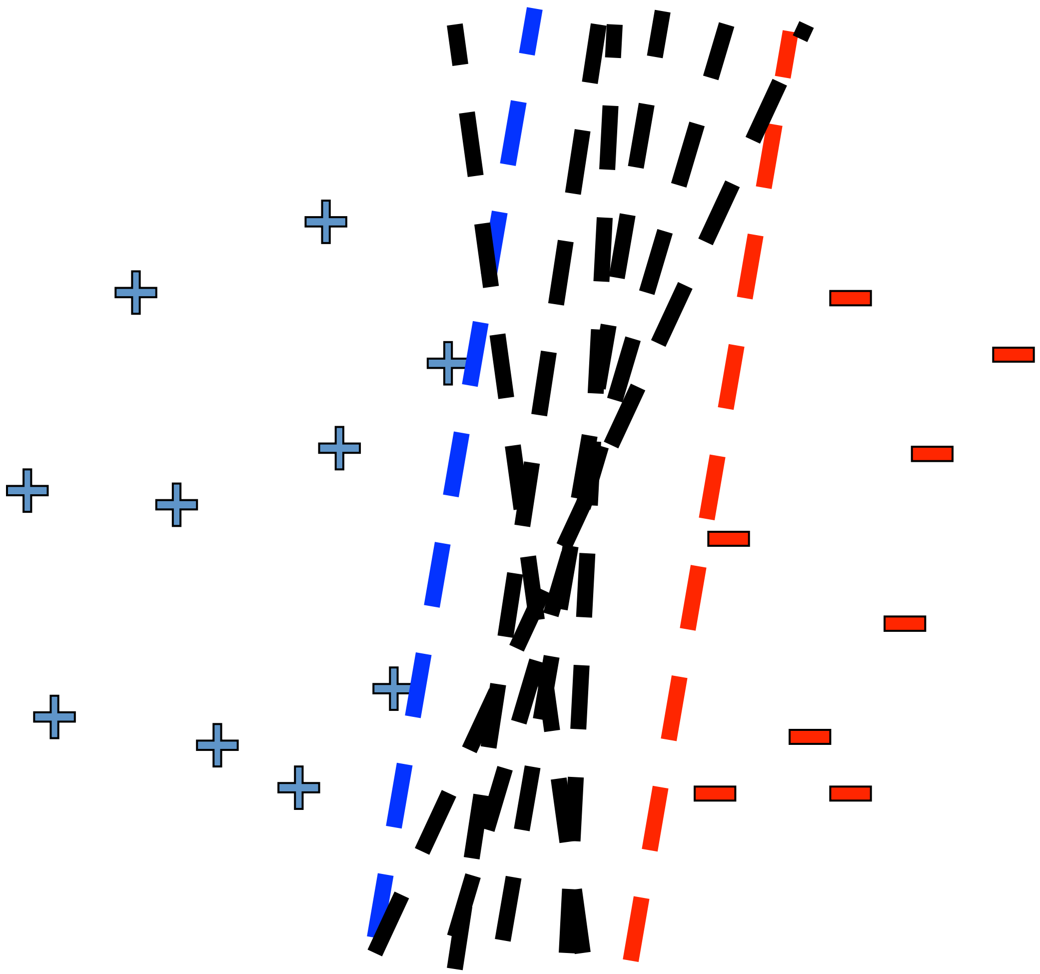
```
function svmModel = trainSVMprimal(traindata,trainlabels,C)

ntrain = size(traindata,1);
ndim = size(traindata,2);

% construct H
H = [illegible];
    [illegible]

% construct f
f = [illegible];

% construct A
A = [illegible]
A = [illegible]
A = [illegible]

% construct b
b = [illegible]

% constuct lb
lb = [illegible]

% solve QP
tic
%z = quadprog(H,f,A,b,[],[],lb);
[z err lm] = qpas(H,f,A,b,[],[],lb);
toc

svmModel.w = [illegible]
svmModel.w0 = [illegible]
```

Administrativia

- Project Mid-Sem Spotlight Presentations
  - Friday: 5-7pm, Whittemore 654
  - 5 slides (recommended)
  - 4 minute time (STRICT) + 1-2 min Q&A
  - Tell the class what you're working on
  - Any results yet?
  - Problems faced?
  - Upload slides on Scholar

Recap of Last Time

Linear classifiers – Which line is better?



$\mathbf{w}.\mathbf{x} = \sum_j w^{(j)} x^{(j)}$

Dual SVM derivation (1) – the linearly separable case

$$\text{minimize}_{\mathbf{w},b} \quad \frac{1}{2}\mathbf{w}.\mathbf{w}$$

$$\left(\mathbf{w}.\mathbf{x}_j + b\right) y_j \geq 1, \ \forall j$$

Dual SVM derivation (1) – the linearly separable case

$$L(\mathbf{w}, \alpha) = \frac{1}{2}\mathbf{w}.\mathbf{w} - \sum_j \alpha_j \left[\left(\mathbf{w}.\mathbf{x}_j + b\right) y_j - 1\right]$$
$$\alpha_j \geq 0, \ \forall j$$

$$\mathbf{w} = \sum_j \alpha_j y_j \mathbf{x}_j$$

Dual SVM formulation – the linearly separable case

$$\text{maximize}_\alpha \quad \sum_i \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i\alpha_j y_i y_j \mathbf{x}_i \mathbf{x}_j$$

$$\sum_i \alpha_i y_i = 0$$
$$\alpha_i \geq 0$$

$$\boxed{\begin{array}{l} \mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i \\ b = y_k - \mathbf{w}.\mathbf{x}_k \\ \text{for any } k \text{ where } \alpha_k > 0 \end{array}}$$

Dual SVM formulation – the non-separable case

$$\text{minimize}_{\mathbf{w},b} \quad \frac{1}{2}\mathbf{w}.\mathbf{w} + C\sum_j \xi_j$$

$$\left(\mathbf{w}.\mathbf{x}_j + b\right) y_j \geq 1 - \xi_j, \ \ \forall j$$

$$\xi_j \geq 0, \ \ \forall j$$

Dual SVM formulation – the non-separable case

$$\text{maximize}_\alpha \quad \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j \mathbf{x}_i \mathbf{x}_j$$

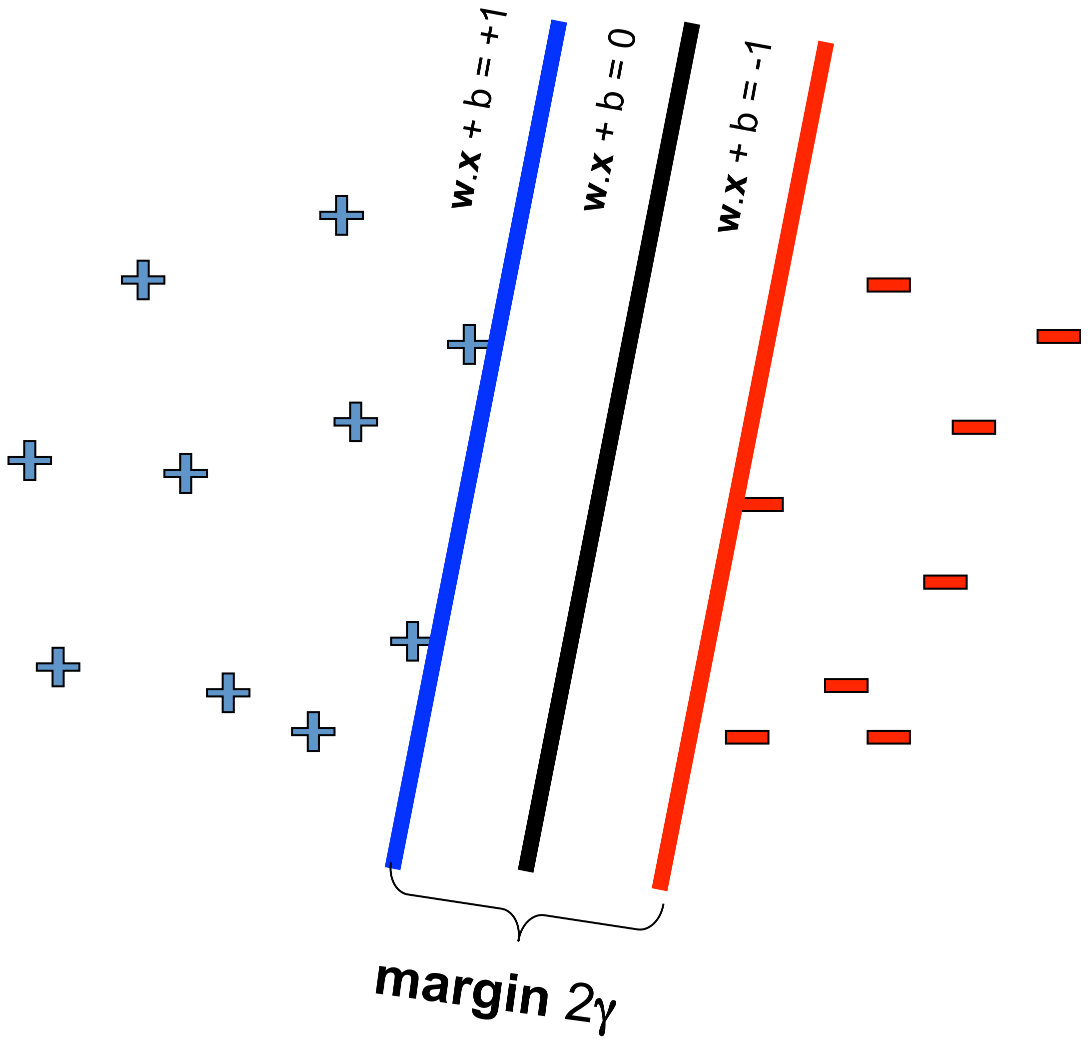
$$\sum_i \alpha_i y_i = 0$$
$$C \geq \alpha_i \geq 0$$

$$\boxed{\begin{gathered} \mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i \\ b = y_k - \mathbf{w}.\mathbf{x}_k \\ \text{for any } k \text{ where } C > \alpha_k > 0 \end{gathered}}$$

Why did we learn about the dual SVM?

- Builds character!
- Exposes structure about the problem
- There are some quadratic programming algorithms that can solve the dual faster than the primal
- The "**kernel trick**"!!!

Dual SVM interpretation: Sparsity



$$\mathbf{w} = \sum_j \alpha_j y_j \mathbf{x}_j$$

Dual formulation only depends on dot-products, not on **w**!

$$\text{maximize}_\alpha \quad \sum_i \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i \alpha_j y_i y_j \mathbf{x}_i \mathbf{x}_j$$

$$\sum_i \alpha_i y_i = 0$$
$$C \geq \alpha_i \geq 0$$

$$\text{maximize}_\alpha \quad \sum_i \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i \alpha_j y_i y_j K(\mathbf{x}_i, \mathbf{x}_j)$$

$$K(\mathbf{x}_i, \mathbf{x}_j) = \Phi(\mathbf{x}_i) \cdot \Phi(\mathbf{x}_j)$$
$$\sum_i \alpha_i y_i = 0$$
$$C \geq \alpha_i \geq 0$$

Common kernels

- Polynomials of degree d

$$K(\mathbf{u}, \mathbf{v}) = (\mathbf{u} \cdot \mathbf{v})^d$$

- Polynomials of degree up to d

$$K(\mathbf{u}, \mathbf{v}) = (\mathbf{u} \cdot \mathbf{v} + 1)^d$$

- Gaussian kernel / Radial Basis Function

$$K(\mathbf{u}, \mathbf{v}) = \exp\left(-\frac{||\mathbf{u} - \mathbf{v}||^2}{2\sigma^2}\right)$$

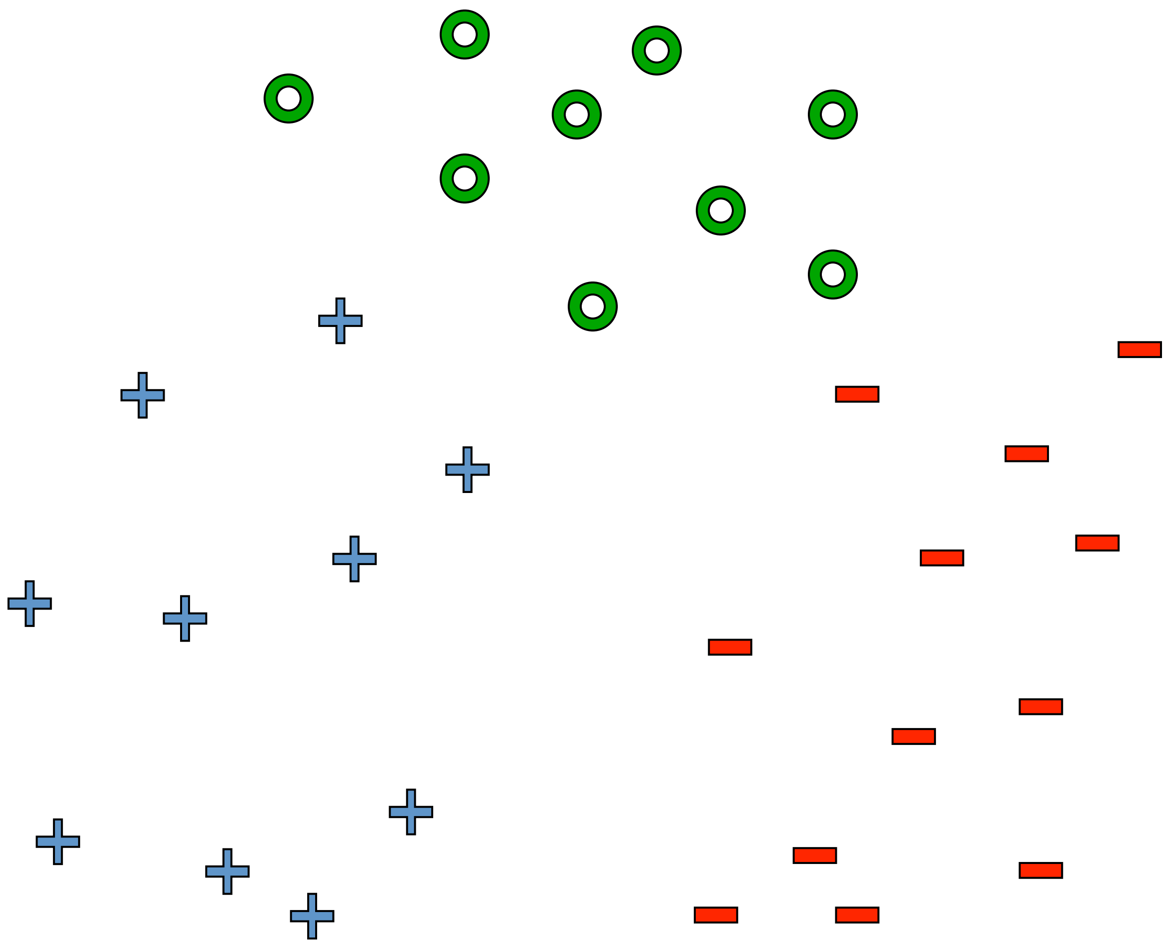
- Sigmoid

$$K(\mathbf{u}, \mathbf{v}) = \tanh(\eta \mathbf{u} \cdot \mathbf{v} + \nu)$$

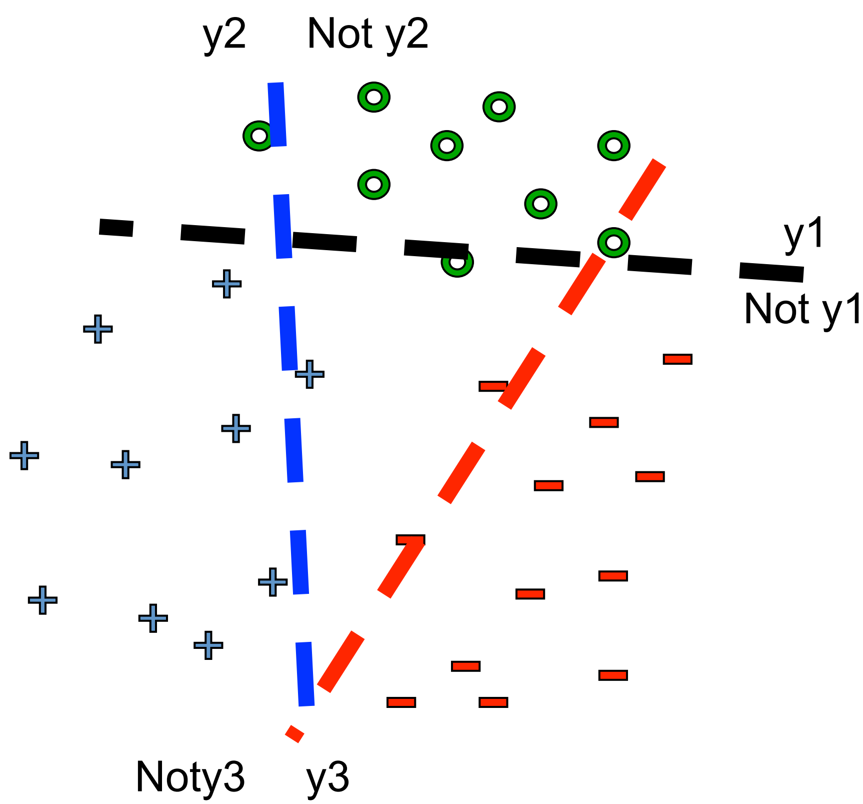
Plan for Today

- SVMs
  - Multi-class
- Neural Networks

What about multiple classes?

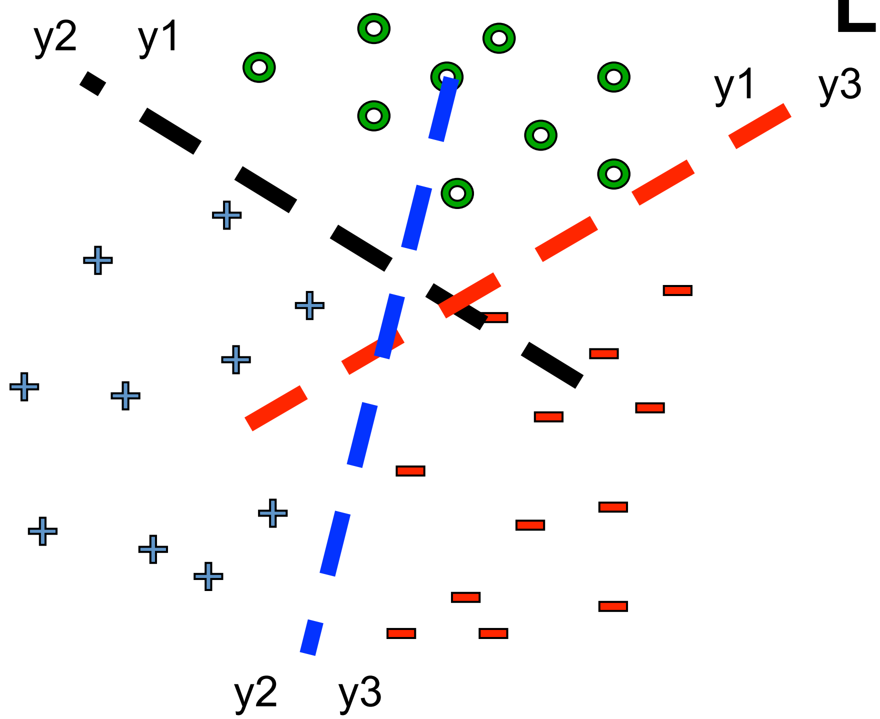


One against All (Rest)



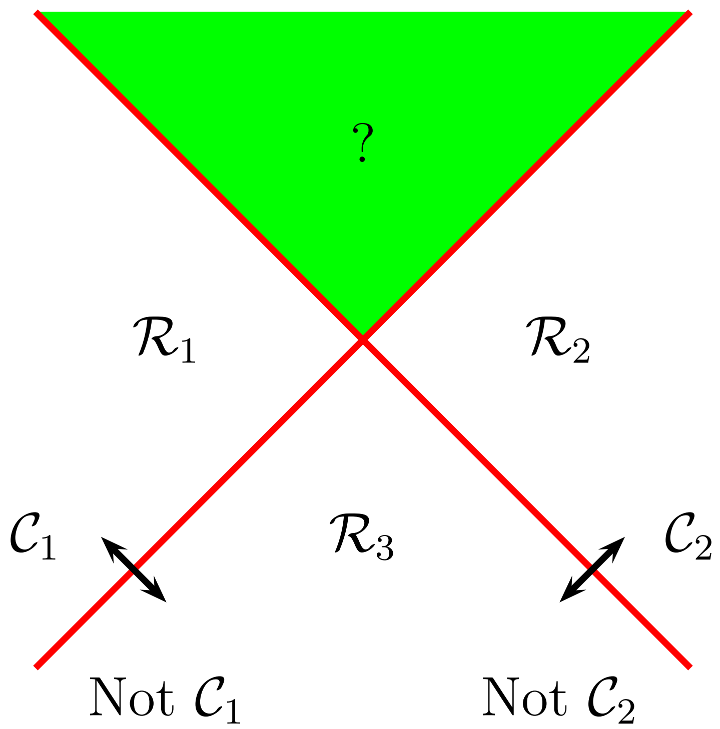
Learn N classifiers:

One against One

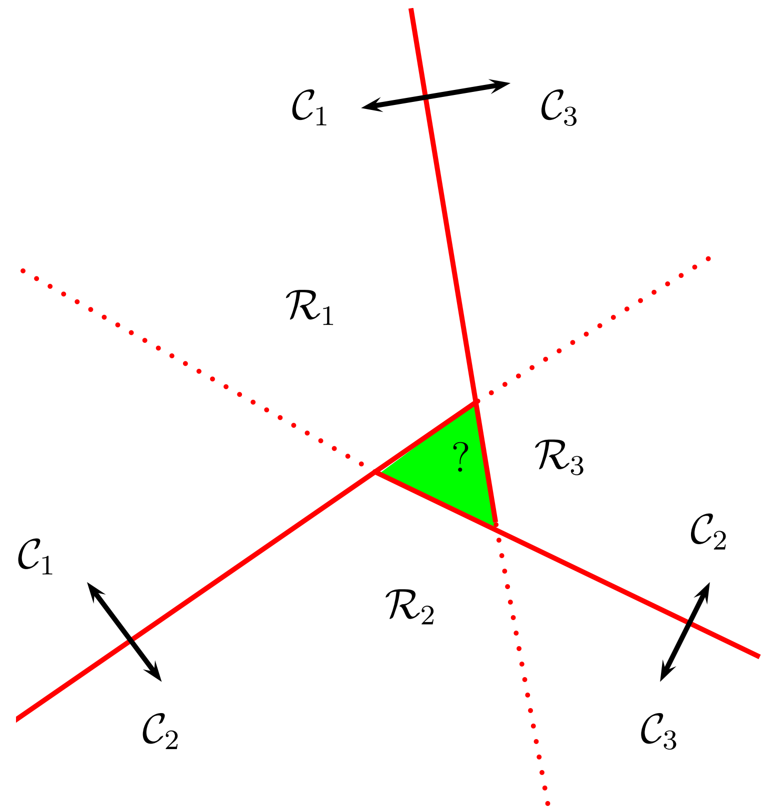


Learn N-choose-2 classifiers:

Problems



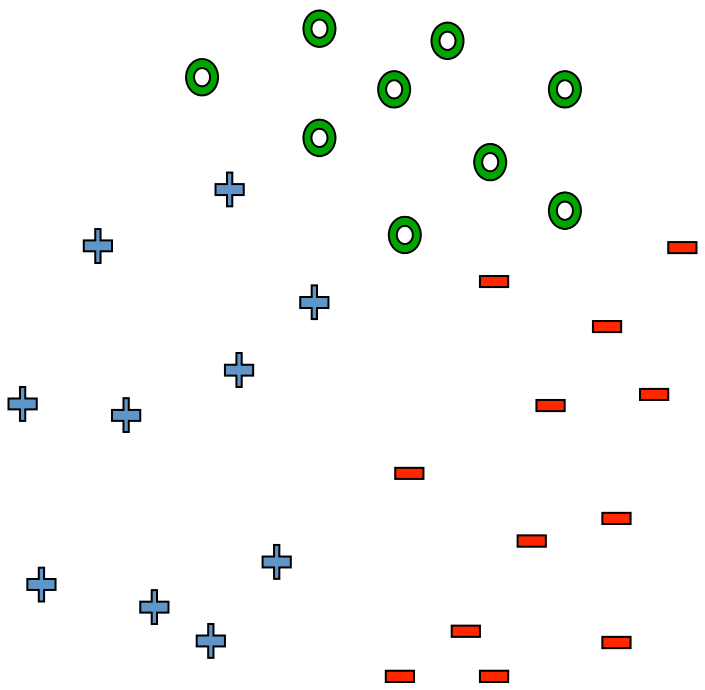
?
$\mathcal{R}_1$
$\mathcal{R}_2$
$\mathcal{C}_1$
$\mathcal{R}_3$
$\mathcal{C}_2$
Not $\mathcal{C}_1$
Not $\mathcal{C}_2$



$\mathcal{C}_1$
$\mathcal{C}_3$
$\mathcal{R}_1$
?
$\mathcal{R}_3$
$\mathcal{C}_2$
$\mathcal{C}_1$
$\mathcal{R}_2$
$\mathcal{C}_2$
$\mathcal{C}_3$

Learn 1 classifier: Multiclass SVM

Simultaneously learn 3 sets of weights



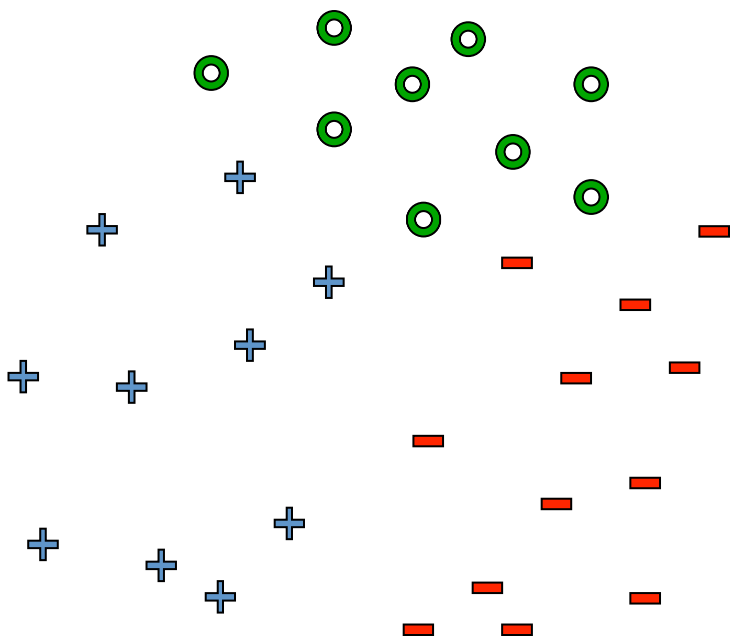
$$\mathbf{w}^{(y_j)}.\mathbf{x}_j + b^{(y_j)} \geq \mathbf{w}^{(y')}.\mathbf{x}_j + b^{(y')} + 1,\ \forall y' \neq y_j,\ \forall j$$

Learn 1 classifier: Multiclass SVM

$$\text{minimize}_{\mathbf{w},b} \quad \sum_y \mathbf{w}^{(y)}.\mathbf{w}^{(y)} + C\sum_j \xi_j$$

$$\mathbf{w}^{(y_j)}.\mathbf{x}_j + b^{(y_j)} \geq \mathbf{w}^{(y')}.\mathbf{x}_j + b^{(y')} + 1 - \xi_j, \;\; \forall y' \neq y_j, \;\; \forall j$$

$$\xi_j \geq 0, \;\; \forall j$$



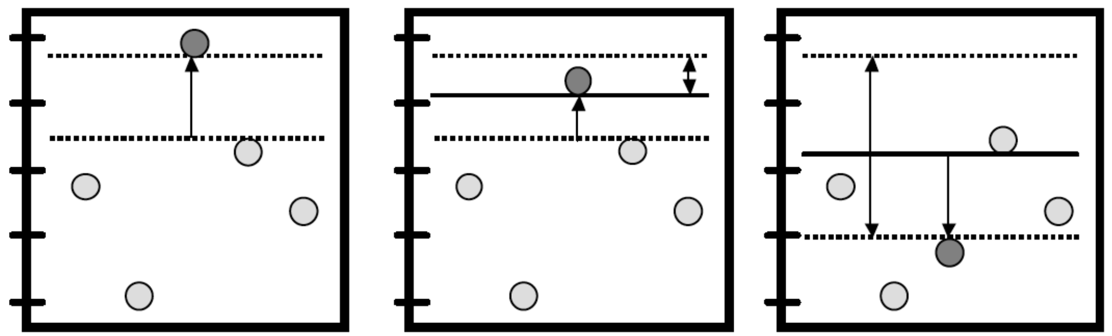


Figure 1: Illustration of the margin bound employed by the optimization problem.

Not linearly separable data

- Some datasets are **not linearly separable!**
  - http://www.eee.metu.edu.tr/~alatan/Courses/Demo/AppletSVM.html

Addressing non-linearly separable data – Option 1, non-linear features

- Choose non-linear features, e.g.,
  - Typical linear features: $w_0 + \sum_i w_i x_i$
  - Example of non-linear features:
    - Degree 2 polynomials, $w_0 + \sum_i w_i x_i + \sum_{ij} w_{ij} x_i x_j$

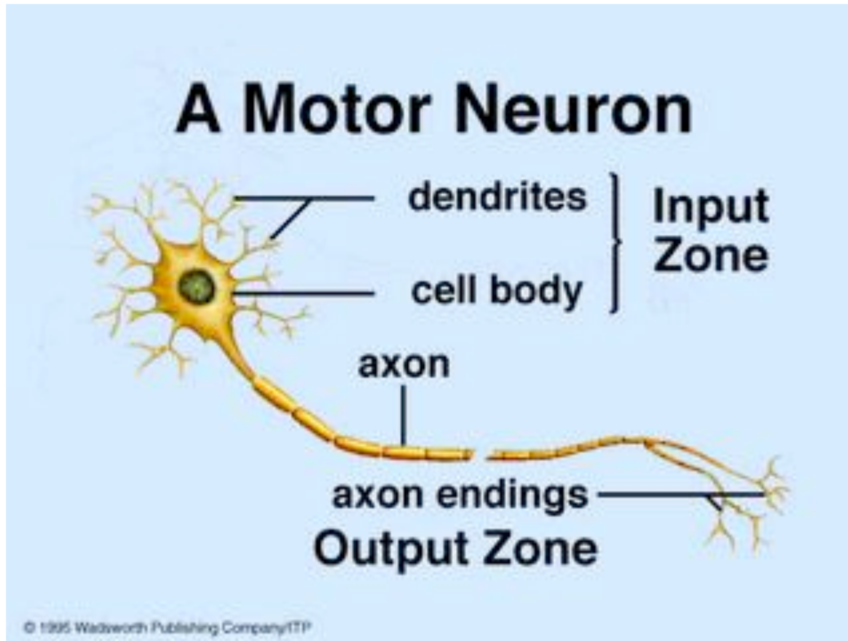
- Classifier $h_{\mathbf{w}}(\mathbf{x})$ still linear in parameters **w**
  - As easy to learn
  - Data is linearly separable in higher dimensional spaces
  - Express via kernels

Addressing non-linearly separable data – Option 2, non-linear classifier

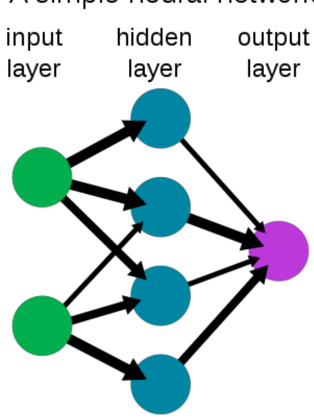
- Choose a classifier $h_{\mathbf{w}}(\mathbf{x})$ that is non-linear in parameters **w**, e.g.,
  - Decision trees, neural networks,…

- More general than linear classifiers
- But, can often be harder to learn (non-convex optimization required)
- Often very useful (outperforms linear classifiers)
- In a way, both ideas are related

New Topic: Neural Networks



A simple neural network



Synonyms

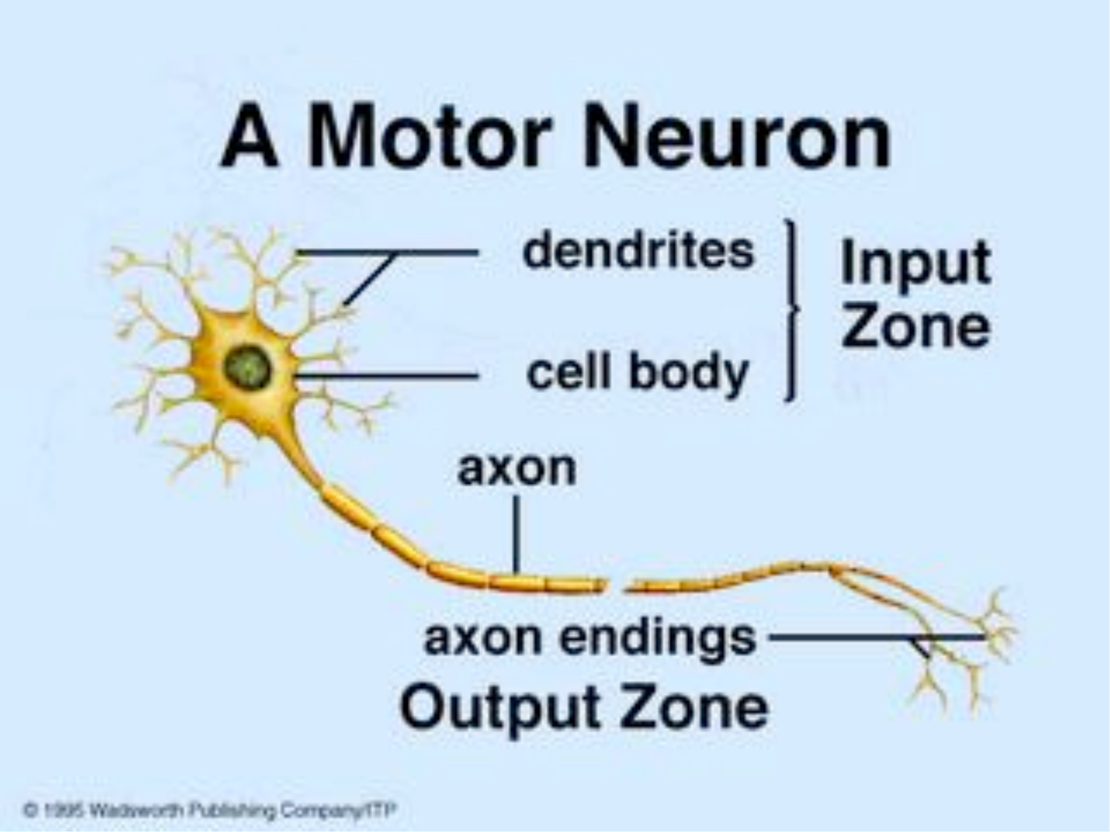
- Neural Networks

- Artificial Neural Network (ANN)
- Feed-forward Networks
- Multilayer Perceptrons (MLP)

- Types of ANN
  – Convolutional Nets
  – Autoencoders
  – Recurrent Neural Nets

- [Back with a new name]: Deep Nets / Deep Learning

Biological Neuron



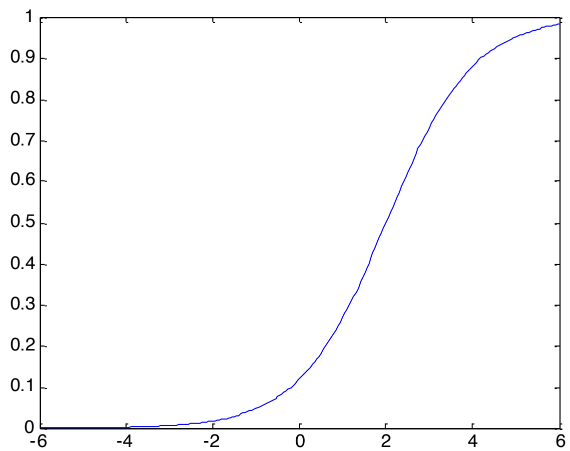
Artificial "Neuron"

- Perceptron (with step function)
- Logistic Regression (with sigmoid)

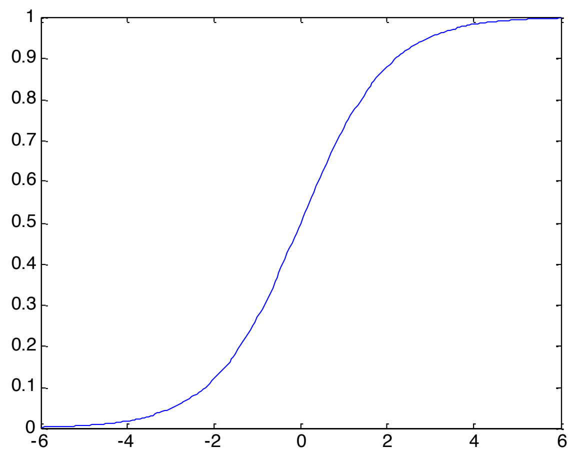
Sigmoid

$$g(w_0 + \sum_i w_i x_i) = \frac{1}{1 + e^{-(w_0 + \sum_i w_i x_i)}}$$

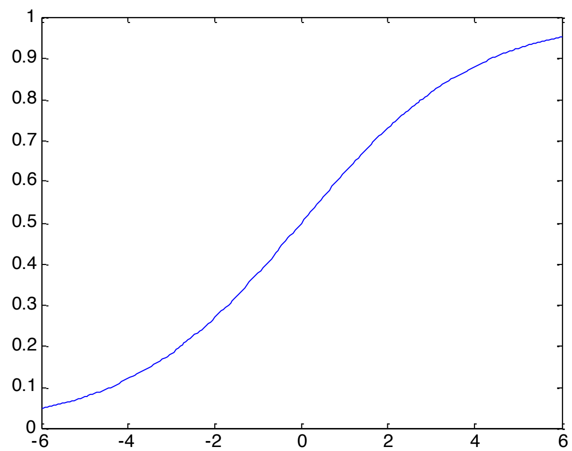
w_0=2, w_1=1



w_0=0, w_1=1



w_0=0, w_1=0.5



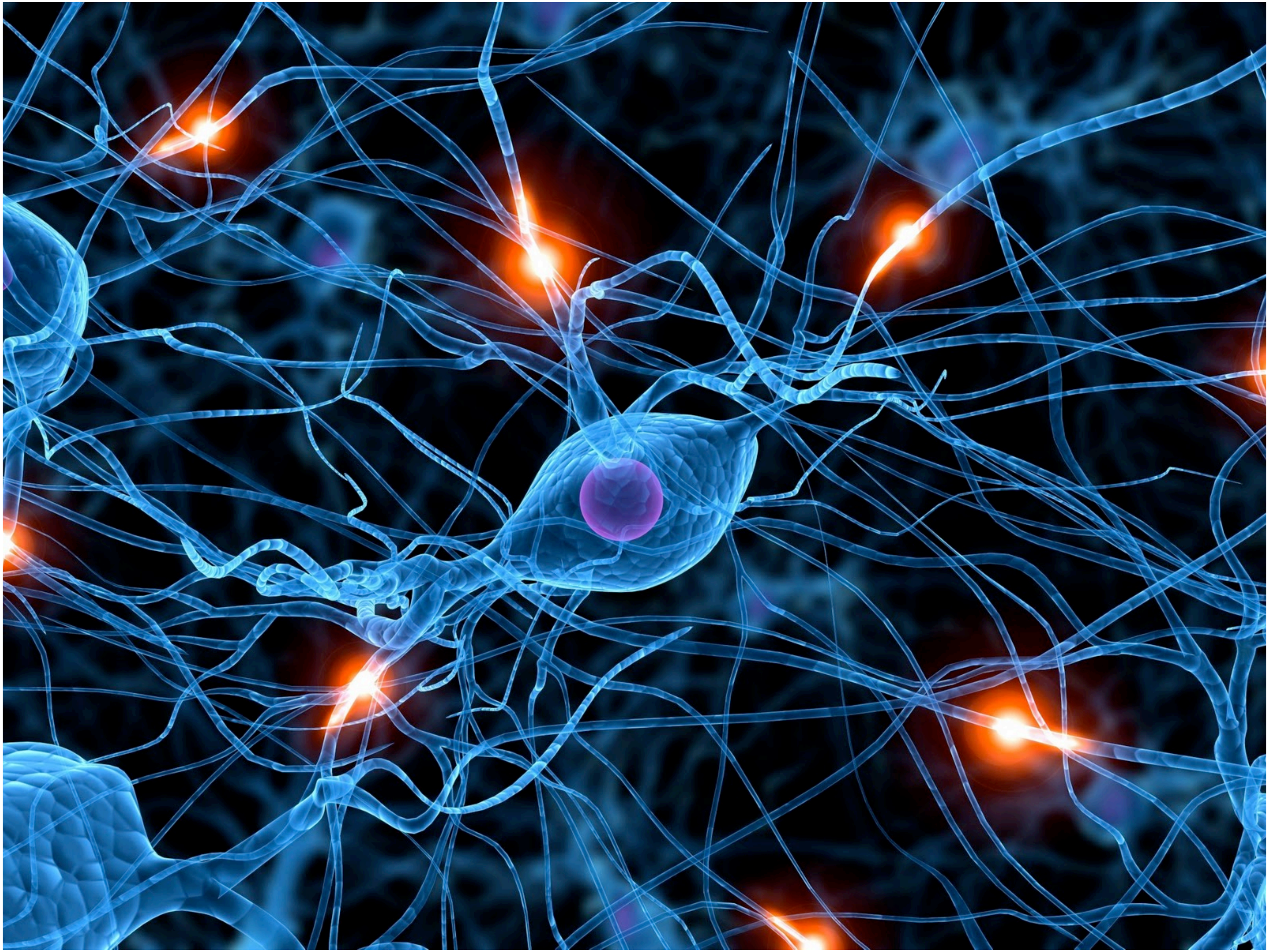
Many possible response functions

- Linear
- Sigmoid
- Exponential
- Gaussian
- …

Limitation

- A single “neuron” is still a linear decision boundary

- What to do?



Limitation

- A single “neuron” is still a linear decision boundary

- What to do?

- Idea: Stack a bunch of them together!

Hidden layer

- 1-hidden layer (or 3-layer network):
  - On board

Neural Nets

- Best performers on OCR
  - http://yann.lecun.com/exdb/lenet/index.html
- NetTalk
  - Text to Speech system from 1987
  - http://youtu.be/tXMaFhO6dIY?t=45m15s
- Rick Rashid speaks Mandarin
  - http://youtu.be/Nu-nlQqFCKg?t=7m30s

Universal Function Approximators

- Theorem
  - 3-layer network with linear outputs can uniformly approximate any continuous function to arbitrary accuracy, given enough hidden units [Funahashi ’89]

Neural Networks

- Demo
  - http://neuron.eng.wayne.edu/bpFunctionApprox/bpFunctionApprox.html